# Package: dynConfiR (via r-universe)

August 26, 2024

```
Type Package
Title Dynamic Models for Confidence and Response Time Distributions
Version 0.1.0
Maintainer Sebastian Hellmann < sebastian.hellmann@ku.de>
Description Provides density functions for the joint distribution of
     choice, response time and confidence for discrete confidence
     judgments as well as functions for parameter fitting,
     prediction and simulation for various dynamical models of
     decision confidence. All models are explained in detail by
     Hellmann et al. (2023; Preprint available at
     <a href="https://osf.io/9jfqr/">, published version:</a>
     <doi:10.1037/rev0000411>). Implemented models are the dynaViTE
     model, dynWEV model, the 2DSD model (Pleskac & Busemeyer, 2010,
     <doi:10.1037/a0019737>), and various race models. C++ code for
     dynWEV and 2DSD is based on the 'rtdists' package by Henrik
     Singmann.
License GPL (>= 3)
URL https://github.com/SeHellmann/dynConfiR,
     https://sehellmann.github.io/dynConfiR/
BugReports https://github.com/SeHellmann/dynConfiR
Depends R (>= 4.0)
Imports dplyr, magrittr, minqa, parallel, progress, Rcpp, rlang, stats
Suggests covr, ggplot2, Hmisc, knitr, logger, rmarkdown, testthat (>=
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2 dynConfiR-package

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## Description

Dynamic Models for Confidence and Response Time Distributions

ConfidenceOrientation 3

#### **Details**

Package: dynConfiR
Type: Package
Version: 0.1.0
Date: 2023-06-19
Depends: R (>= 4.0)
License: GPL (>=3)

URL: https://github.com/SeHellmann/dynConfiR

Provides response time and confidence distributions (density/PDF) for following models: dynaViTE,dynWEV, 2DSD, 2DSDT, IRM and PCRM

#### Author(s)

Sebastian Hellmann

ConfidenceOrientation Confidence and response time data

## **Description**

A data set containing results from an orientation discrimination experiment with confidence judgments. The data set includes results from 16 participants and 3 sessions. The task was to identify the orientation (horizontal or vertical) of a grid that was briefly visible and then covered by a mask in form of a checkerboard pattern.

#### Usage

data(ConfidenceOrientation)

#### **Format**

A data frame with 25920 rows and 12 variables:

participant integer values as unique participant identifier

session session identifier ranging from 1 to 3

gender gender of the participant: "w" for female; "m" for male participants

age the age of participants in years

SOA stimulus-onset-asynchrony in ms (i.e. time between stimulus and mask onset)

orientation orientation of the target stimulus (0: vertical, 90: horizontal)

stimulus stimulus identity ("senkrecht": vertical, "waagrecht": horizontal)

response response for the discrimination task (see stimulus column)

correct 0-1 column indicating whether the discrimination response was correct (1) or not (0)

rt response time for the discrimination response in sec

cont\_rating confidence rating as registered (continuous values ranging from -1 (unsure) to 1 (sure))

disc\_rating confidence rating discretized in 5 steps using equidistant breaks

#### **Source**

https://github.com/SeHellmann/SeqSamplingConfidenceModels

d2DSD

Pleskac and Busemeyer's 2DSD Model for Decision Confidence

## Description

Likelihood function and random number generator for a generalization of the 2DSD Model presented by Pleskac & Busemeyer (2010). It includes following parameters: DDM parameters: a (threshold separation), z (starting point; relative), v (drift rate), t0 (non-decision time/ response time constant), d (differences in speed of response execution), sv (inter-trial-variability of drift), st0 (inter-trial-variability of non-decisional components), sz (inter-trial-variability of relative starting point), s (diffusion constant).

## Usage

```
d2DSD(rt, response = "upper", th1, th2, a, v, t0 = 0, z = 0.5, d = 0,
    sz = 0, sv = 0, st0 = 0, tau = 1, lambda = 0, s = 1,
    simult_conf = FALSE, precision = 6, z_absolute = FALSE,
    stop_on_error = TRUE, stop_on_zero = FALSE)

r2DSD(n, a, v, t0 = 0, z = 0.5, d = 0, sz = 0, sv = 0, st0 = 0,
    tau = 1, lambda = 0, s = 1, delta = 0.01, maxrt = 15,
    simult_conf = FALSE, z_absolute = FALSE, stop_on_error = TRUE)
```

#### **Arguments**

а

rt	a vector of RTs. Or for convenience also a data.frame with columns ${\sf rt}$ and ${\sf response}$ .
response	character vector, indicating the decision, i.e. which boundary was met first. Possible values are c("upper", "lower") (possibly abbreviated) and "upper" being the default. Alternatively, a numeric vector with values 1=lower and 2=upper or -1=lower and 1=upper, respectively. For convenience, response is converted via as.numeric also allowing factors. Ignored if the first argument is a data.frame.
th1	together with th2: scalars or numerical vectors giving the lower and upper bound of the interval, in which the accumulator should end at the time of the confidence judgment (i.e. at time rt+tau). Only values with th2>=th1 are accepted.

th2 (see th1)

threshold separation. Amount of information that is considered for a decision. Large values indicate a conservative decisional style. Typical range: 0.5 < a < 2

v	drift rate. Average slope of the information accumulation process. The drift gives information about the speed and direction of the accumulation of information. Large (absolute) values of drift indicate a good performance. If received information supports the response linked to the upper threshold the sign will be positive and vice versa. Typical range: $-5 < v < 5$
t0	non-decision time or response time constant (in seconds). Lower bound for the duration of all non-decisional processes (encoding and response execution). Typical range: $0.1 < t0 < 0.5$ . Default is 0.
Z	(by default relative) starting point. Indicator of an a priori bias in decision making. When the relative starting point z deviates from $\emptyset.5$ , the amount of information necessary for a decision differs between response alternatives. Default is $\emptyset.5$ (i.e., no bias).
d	differences in speed of response execution (in seconds). Positive values indicate that response execution is faster for responses linked to the upper threshold than for responses linked to the lower threshold. Typical range: $-0.1 < d < 0.1$ . Default is 0.
SZ	inter-trial-variability of starting point. Range of a uniform distribution with mean z describing the distribution of actual starting points from specific trials. Values different from 0 can predict fast errors (but can slow computation considerably). Typical range: $0 < sz < 0.2$ . Default is 0. (Given in relative range i.e. bounded by $2*min(z, 1-z)$ )
sv	inter-trial-variability of drift rate. Standard deviation of a normal distribution with mean $\nu$ describing the distribution of actual drift rates from specific trials. Values different from 0 can predict slow errors. Typical range: $0 < s\nu < 2$ . Default is 0.
st0	inter-trial-variability of non-decisional components. Range of a uniform distribution with mean $t0 + st0/2$ describing the distribution of actual $t0$ values across trials. Accounts for response times below $t0$ . Reduces skew of predicted RT distributions. Values different from 0 can slow computation considerably. Typical range: $0 < st0 < 0.2$ . Default is 0.
tau	post-decisional accumulation time. The length of the time period after the decision was made until the confidence judgment is made. Range: tau>0. Default: tau=1.
lambda	power for judgment time in the division of the confidence measure by the judgment time (Default: 0, i.e. no division which is the version of 2DSD proposed by Pleskac and Busemeyer)
s	diffusion constant. Standard deviation of the random noise of the diffusion process (i.e., within-trial variability), scales a, v, sv, and th's. Needs to be fixed to a constant in most applications. Default is 1. Note that the default used by Ratcliff and in other applications is often 0.1.
simult_conf	logical. Whether in the experiment confidence was reported simultaneously with the decision, as then decision and confidence judgment are assumed to have happened subsequent before response and computations are different, when there is

an observable interjudgment time (then simult\_conf should be FALSE).

precision	numerical scalar value. Precision of calculation. Determines the the stepsize of integration w.r.t. z and t0. Represents the number of decimals precisely computet on average. Default is 6.
z_absolute	logical. Determines whether $z$ is treated as absolute start point (TRUE) or relative (FALSE; default) to a.
stop_on_error	Should the diffusion functions return $0$ if the parameters values are outside the allowed range (= FALSE) or produce an error in this case (= TRUE).
stop_on_zero	Should the computation of densities stop as soon as a density value of 0 occurs. This may save a lot of time if the function is used for a likelihood function. Default: FALSE
n	integer. The number of samples generated.
delta	numeric. Discretization step size for simulations in the stochastic process
maxrt	numeric. Maximum decision time returned. If the simulation of the stochastic process exceeds a decision time of maxrt, the response will be set to 0 and the maxrt will be returned as rt.

#### **Details**

For confidence: tau (post-decisional accumulation time), lambda the exponent of judgment time for the division by judgment time in the confidence measure, th1 and th2 (lower and upper thresholds for confidence interval).

# Note that the parameterization or defaults of non-decision time variability st0 and diffusion constant s differ from what is often found in the literature.

The drift diffusion model (DDM; Ratcliff and McKoon, 2008) is a mathematical model for two-choice discrimination tasks. It is based on the assumption that information is accumulated continuously until one of two decision thresholds is hit. For introduction see Ratcliff and McKoon (2008).

The 2DSD is an extension of the DDM to explain confidence judgments based on the preceding decision. It assumes a post decisional period where the process continues the accumulation of information. At the end of the period a confidence judgment (i.e. a judgment of the probability that the decision was correct) is made based on the state of the process. Here, we use a given interval, given by th1 and th2, assuming that the data is given with discrete judgments and pre-processed, s.t. these discrete ratings are translated to the respective intervals. The 2DSD Model was proposed by Pleskac and Busemeyer (2010).

All functions are fully vectorized across all parameters as well as the response to match the length or rt (i.e., the output is always of length equal to rt). This allows for trial wise parameters for each model parameter.

For convenience, the function allows that the first argument is a data.frame containing the information of the first and second argument in two columns (i.e., rt and response). Other columns (as well as passing response separately argument) will be ignored.

#### Value

d2DSD gives the density/likelihood/probability of the diffusion process producing a decision of response at time rt and a confidence judgment corresponding to the interval [ th1, th2]. The value will be a numeric vector of the same length as rt.

r2DSD returns a data.frame with three columns and n rows. Column names are rt (response time), response (-1 (lower) or 1 (upper), indicating which bound was hit), and conf (the value of the confidence measure; not discretized!).

The distribution parameters (as well as response, tau, th1 and th2) are recycled to the length of the result. In other words, the functions are completely vectorized for all parameters and even the response boundary.

#### Note

The parameterization of the non-decisional components, t0 and st0, differs from the parameterization sometimes used in the literature. In the present case t0 is the lower bound of the uniform distribution of length st0, but *not* its midpoint. The parameterization employed here is in line with the functions in the rtdists package.

The default diffusion constant s is 1 and not 0.1 as in most applications of Roger Ratcliff and others. Usually s is not specified as the other parameters: a, v, and sv, may be scaled to produce the same distributions (as is done in the code).

The function code is basically an extension of the ddiffusion function from the package rtdists for the Ratcliff diffusion model.

#### Author(s)

For the original rtdists package: Underlying C code by Jochen Voss and Andreas Voss. Porting and R wrapping by Matthew Gretton, Andrew Heathcote, Scott Brown, and Henrik Singmann. qdiffusion by Henrik Singmann. For the d2DSD function the C code was extended by Sebastian Hellmann.

## References

Pleskac, T. J., & Busemeyer, J. R. (2010). Two-Stage Dynamic Signal Detection: A Theory of Choice, Decision Time, and Confidence, *Psychological Review*, 117(3), 864-901. doi:10.1037/a0019737

Ratcliff, R., & McKoon, G. (2008). The diffusion decision model: Theory and data for two-choice decision tasks. *Neural Computation*, 20(4), 873-922.

#### **Examples**

```
# Plot rt distribution ignoring confidence
curve(d2DSD(x, "upper", -Inf, Inf, tau=1, a=2, v=0.4, sz=0.2, sv=0.9), xlim=c(0, 2), lty=2)
curve(d2DSD(x, "lower", -Inf, Inf, tau=1, a=2, v=0.4, sz=0.2, sv=0.9), col="red", lty=2, add=TRUE)
curve(d2DSD(x, "upper", -Inf, Inf, tau=1, a=2, v=0.4), add=TRUE)
curve(d2DSD(x, "lower", -Inf, Inf, tau=1, a=2, v=0.4), col="red", add=TRUE)
# Generate a random sample
dfu <- r2DSD(5000, a=2,v=0.5,t0=0,z=0.5,d=0,sz=0,sv=0, st0=0, tau=1, s=1)
# Same RT distribution but upper and lower responses changed
dfl <- r2DSD(50, a=2,v=-0.5,t0=0,z=0.5,d=0,sz=0,sv=0, st0=0, tau=1, s=1)
head(dfu)

d2DSD(dfu, th1=-Inf, th2=Inf, a=2, v=.5)[1:5]
# Scaling diffusion parameters leads do same density values
s <- 2</pre>
```

```
d2DSD(dfu, th1=-Inf, th2=Inf, a=2*s, v=.5*s, s=2)[1:5]
if (requireNamespace("ggplot2", quietly = TRUE)) {
    require(ggplot2)
    ggplot(dfu, aes(x=rt, y=conf))+
        stat_density_2d(aes(fill = after_stat(density)), geom = "raster", contour = FALSE) +
        facet_wrap(~response)
}
boxplot(conf~response, data=dfu)

# Restricting to specific confidence region
dfu <- dfu[dfu$conf >0 & dfu$conf <1,]
d2DSD(dfu, th1=0, th2=1, a=2, v=0.5)[1:5]

# If lower confidence threshold is higher than the upper, the function throws an error,
# except when stop_on_error is FALSE
d2DSD(dfu[1:5,], th1=1, th2=0, a=2, v=0.5, stop_on_error = FALSE)</pre>
```

dDDConf

Drift Diffusion Model with time-dependent confidence

## Description

Likelihood function and random number generator for the Drift Diffusion Model with confidence computed as decision time. It includes following parameters: DDM parameters: a (threshold separation), z (starting point; relative), v (drift rate), t0 (non-decision time/ response time constant), d (differences in speed of response execution), sv (inter-trial-variability of drift), st0 (inter-trial-variability of non-decision components), sz (inter-trial-variability of relative starting point), s (diffusion constant).

#### Usage

```
dDDConf(rt, response = "upper", th1, th2, a, v, t0 = 0, z = 0.5, d = 0,
    sz = 0, sv = 0, st0 = 1, s = 1, precision = 6,
    z_absolute = FALSE, stop_on_error = TRUE, stop_on_zero = FALSE,
    st0stepsize = 0.001)

rDDConf(n, a, v, t0 = 0, z = 0.5, d = 0, sz = 0, sv = 0, st0 = 2,
    s = 1, delta = 0.01, maxrt = 15, z_absolute = FALSE,
    stop_on_error = TRUE)
```

#### Arguments

rt a vector of RTs. Or for convenience also a data.frame with columns rt and

response.

response character vector, indicating the decision, i.e. which boundary was met first. Possible values are c("upper", "lower") (possibly abbreviated) and "upper"

	being the default. Alternatively, a numeric vector with values 1=lower and 2=upper or -1=lower and 1=upper, respectively. For convenience, response is converted via as.numeric also allowing factors. Ignored if the first argument is a data.frame.
th1	together with th2: scalars or numerical vectors giving the lower and upper bound of the interval, in which the accumulator should end at the time of the confidence judgment (i.e. at time rt+tau). Only values with th2>=th1 are accepted.
th2	(see th1)
a	threshold separation. Amount of information that is considered for a decision. Large values indicate a conservative decisional style. Typical range: $0.5 < a < 2$
V	drift rate. Average slope of the information accumulation process. The drift gives information about the speed and direction of the accumulation of information. Large (absolute) values of drift indicate a good performance. If received information supports the response linked to the upper threshold the sign will be positive and vice versa. Typical range: $-5 < v < 5$
t0	non-decision time or response time constant (in seconds). Lower bound for the duration of all non-decisional processes (encoding and response execution). Typical range: $0.1 < t0 < 0.5$ . Default is 0.
z	(by default relative) starting point. Indicator of an a priori bias in decision making. When the relative starting point z deviates from $\emptyset.5$ , the amount of information necessary for a decision differs between response alternatives. Default is $\emptyset.5$ (i.e., no bias).
d	differences in speed of response execution (in seconds). Positive values indicate that response execution is faster for responses linked to the upper threshold than for responses linked to the lower threshold. Typical range: $-0.1 < d < 0.1$ . Default is 0.
SZ	inter-trial-variability of starting point. Range of a uniform distribution with mean z describing the distribution of actual starting points from specific trials. Values different from 0 can predict fast errors (but can slow computation considerably). Typical range: $0 < sz < 0.2$ . Default is 0. (Given in relative range i.e. bounded by $2*min(z, 1-z)$ )
SV	inter-trial-variability of drift rate. Standard deviation of a normal distribution with mean $\nu$ describing the distribution of actual drift rates from specific trials. Values different from 0 can predict slow errors. Typical range: $0 < s\nu < 2$ . Default is 0.
st0	inter-trial-variability of non-decisional components. Range of a uniform distribution with mean $t0 + st0/2$ describing the distribution of actual $t0$ values across trials. Accounts for response times below $t0$ . Reduces skew of predicted RT distributions. Values different from 0 can slow computation considerably. Typical range: $0 < st0 < 0.2$ . Default is 0.
s	diffusion constant. Standard deviation of the random noise of the diffusion process (i.e., within-trial variability), scales a, v, sv, and th's. Needs to be fixed to a constant in most applications. Default is 1. Note that the default used by

Ratcliff and in other applications is often 0.1.

precision numerical scalar value. Precision of calculation. Corresponds to the stepsize of integration w.r.t. z. Default is 1e-5. z\_absolute logical. Determines whether z is treated as absolute start point (TRUE) or relative (FALSE; default) to a. Should the diffusion functions return 0 if the parameters values are outside the stop\_on\_error allowed range (= FALSE) or produce an error in this case (= TRUE). stop\_on\_zero Should the computation of densities stop as soon as a density value of 0 occurs. This may save a lot of time if the function is used for a likelihood function. Default: FALSE numerical scalar value. Stepsize for integration over t0. st0stepsize integer. The number of samples generated. delta numeric. Discretization step size for simulations in the stochastic process numeric. Maximum decision time returned. If the simulation of the stochastic maxrt process exceeds a decision time of maxrt, the response will be set to 0 and the maxrt will be returned as rt.

#### **Details**

For the confidence part: th1 and th2 (lower and upper thresholds for decision time interval).

Note that the parameterization or defaults of non-decision time variability st0 and diffusion constant s differ from what is often found in the literature.

The Ratcliff diffusion model (Ratcliff and McKoon, 2008) is a mathematical model for two-choice discrimination tasks. It is based on the assumption that information is accumulated continuously until one of two decision thresholds is hit. For introduction see Ratcliff and McKoon (2008).

This model incorporates the idea, that the decision time T is informative for stimulus difficulty and thus confidence is computed as a monotone function of  $\frac{1}{\sqrt{T}}$ . In this implementation, confidence is the decision time, directly. Here, we use an interval, given by th1 and th2, assuming that the data is given with discrete judgments and pre-processed, s.t. these discrete ratings are translated to the respective intervals.

All functions are fully vectorized across all parameters as well as the response to match the length or rt (i.e., the output is always of length equal to rt). This allows for trial wise parameters for each model parameter.

For convenience, the function allows that the first argument is a data.frame containing the information of the first and second argument in two columns (i.e., rt and response). Other columns (as well as passing response separately argument) will be ignored.

## Value

dDDConf gives the density/likelihood/probability of the diffusion process producing a decision of response at time rt and a confidence judgment corresponding to the interval [th1, th2]. The value will be a numeric vector of the same length as rt.

rDDConf returns a data.frame with three columns and n rows. Column names are rt (response time), response (-1 (lower) or 1 (upper), indicating which bound was hit), conf for the decision time (without non-decision time component; not discretized!).

The distribution parameters (as well as response, th1 and th2) are recycled to the length of the result. In other words, the functions are completely vectorized for all parameters and even the response boundary.

#### Note

The parameterization of the non-decisional components, t0 and st0, differs from the parameterization sometimes used in the literature. In the present case t0 is the lower bound of the uniform distribution of length st0, but *not* its midpoint. The parameterization employed here is in line with the functions in the rtdists package.

The default diffusion constant s is 1 and not 0.1 as in most applications of Roger Ratcliff and others. Usually s is not specified as the other parameters: a, v, and sv, may be scaled to produce the same distributions (as is done in the code).

The function code is basically an extension of the ddiffusion function from the package rtdists for the Ratcliff diffusion model.

#### Author(s)

For the original rtdists package: Underlying C code by Jochen Voss and Andreas Voss. Porting and R wrapping by Matthew Gretton, Andrew Heathcote, Scott Brown, and Henrik Singmann. qdiffusion by Henrik Singmann. For the dDDConf function the C code was extended by Sebastian Hellmann.

#### References

Ratcliff, R., & McKoon, G. (2008). The diffusion decision model: Theory and data for two-choice decision tasks. *Neural Computation*, 20(4), 873-922.

Hellmann, S., Zehetleitner, M., & Rausch, M. (2023). Simultaneous modeling of choice, confidence and response time in visual perception. *Psychological Review* 2023 Mar 13. doi: 10.1037/rev0000411. Epub ahead of print. PMID: 36913292.

## Examples

```
# Plot rt distribution ignoring confidence
curve(dDDConf(x, "upper", 0, Inf, a=2, v=0.4, sz=0.2, sv=0.9), xlim=c(0, 2), lty=2)
curve(dDDConf(x, "lower", 0, Inf, a=2, v=0.4, sz=0.2, sv=0.9), col="red", lty=2, add=TRUE)
curve(dDDConf(x, "upper", 0, Inf, a=2, v=0.4), add=TRUE)
curve(dDDConf(x, "lower", 0, Inf, a=2, v=0.4), col="red", add=TRUE)
# Generate a random sample
dfu <- rDDConf(5000, a=2,v=0.5,t0=0,z=0.5,d=0,sz=0,sv=0, st0=2, s=1)
# Same RT distribution but upper and lower responses changed
dfl <- rDDConf(50, a=2,v=-0.5,t0=0,z=0.5,d=0,sz=0,sv=0, st0=2, s=1)
head(dfu)

dDDConf(dfu, th1=0.5, th2=2.5, a=2, v=.5, st0=2)[1:5]
# Scaling diffusion parameters leads do same density values
s <- 2
dDDConf(dfu, th1=0.5, th2=2.5, a=2*s, v=.5*s, s=2, st0=2)[1:5]
if (requireNamespace("ggplot2", quietly = TRUE)) {
    require(ggplot2)</pre>
```

```
ggplot(dfu, aes(x=rt, y=conf))+
    stat_density_2d(aes(fill = after_stat(density)), geom = "raster", contour = FALSE) +
    facet_wrap(~response)
}
boxplot(conf~response, data=dfu)

# Restricting to specific confidence region
dfu <- dfu[dfu$conf >0 & dfu$conf <1,]
dDDConf(dfu, th1=0, th2=1, a=2, v=0.5, st0=2)[1:5]

# If lower confidence threshold is higher than the upper, the function throws an error,
# except when stop_on_error is FALSE
dDDConf(dfu[1:5,], th1=1, th2=0, a=2, v=0.5, stop_on_error = FALSE)</pre>
```

dynaViTE

Dynamical visibility, time, and evidence model (dynaViTE) and Dynamical weighted evidence and visibility model (dynWEV)

#### **Description**

Likelihood function and random number generator for the dynaViTE and dynWEV model (Hellmann et al., 2023). It includes following parameters from the drift diffusion model: a (threshold separation), z (starting point; relative), v (drift rate), t0 (non-decision time/response time constant), d (differences in speed of response execution), sv (inter-trial-variability of drift), st0 (inter-trial-variability of non-decisional components), sz (inter-trial-variability of relative starting point) and s (diffusion constant). For the computation of confidence following parameters were added: tau (post-decisional accumulation time), w (weight on the decision evidence (weight on visibility is (1-w))), muvis (mean drift rate of visibility process), svis (diffusion constant of visibility process), sigvis (variability in drift rate of visibility accumulator), th1 and th2 (lower and upper thresholds for confidence interval). lambda for dynaViTE only, the exponent of judgment time for the division by judgment time in the confidence measure, and Note that the parametrization or defaults of non-decision time variability st0 and diffusion constant s differ from what is often found in the literature.

Likelihood function and random number generator for the dynaViTE and dynWEV model (Hellmann et al., 2023). It includes following parameters from the drift diffusion model: a (threshold separation), z (starting point; relative), v (drift rate), t0 (non-decision time/response time constant), d (differences in speed of response execution), sv (inter-trial-variability of drift), st0 (inter-trial-variability of non-decisional components), sz (inter-trial-variability of relative starting point) and s (diffusion constant). For the computation of confidence following parameters were added: tau (post-decisional accumulation time), w (weight on the decision evidence (weight on visibility is (1-w))), muvis (mean drift rate of visibility process), svis (diffusion constant of visibility process), sigvis (variability in drift rate of visibility accumulator), th1 and th2 (lower and upper thresholds for confidence interval). lambda for dynaViTE only, the exponent of judgment time for the division by judgment time in the confidence measure, and Note that the parametrization or defaults of non-decision time variability st0 and diffusion constant s differ from what is often found in the literature.

#### Usage

```
dWEV(rt, response = "upper", th1, th2, a, v, t0 = 0, z = 0.5, d = 0,
 sz = 0, sv = 0, st0 = 0, tau = 1, w = 0.5, muvis = NULL,
  sigvis = 0, svis = 1, lambda = 0, s = 1, simult_conf = FALSE,
 precision = 6, z_absolute = FALSE, stop_on_error = TRUE,
  stop\_on\_zero = FALSE)
ddynaViTE(rt, response = "upper", th1, th2, a, v, t0 = 0, z = 0.5,
  d = 0, sz = 0, sv = 0, st0 = 0, tau = 1, w = 0.5, muvis = NULL,
  sigvis = 0, svis = 1, lambda = 0, s = 1, simult_conf = FALSE,
  precision = 6, z_absolute = FALSE, stop_on_error = TRUE,
  stop_on_zero = FALSE)
rdynaViTE(n, a, v, t0 = 0, z = 0.5, d = 0, sz = 0, sv = 0, st0 = 0,
  tau = 1, w = 0.5, muvis = NULL, sigvis = 0, svis = 1, lambda = 0,
  s = 1, delta = 0.01, maxrt = 15, simult_conf = FALSE,
 z_absolute = FALSE, stop_on_error = TRUE, process_results = FALSE)
dWEV(rt, response = "upper", th1, th2, a, v, t0 = 0, z = 0.5, d = 0,
  sz = 0, sv = 0, st0 = 0, tau = 1, w = 0.5, muvis = NULL,
  sigvis = 0, svis = 1, lambda = 0, s = 1, simult_conf = FALSE,
 precision = 6, z_absolute = FALSE, stop_on_error = TRUE,
  stop_on_zero = FALSE)
rWEV(n, a, v, t0 = 0, z = 0.5, d = 0, sz = 0, sv = 0, st0 = 0,
  tau = 1, w = 0.5, muvis = NULL, sigvis = 0, svis = 1, lambda = 0,
  s = 1, delta = 0.01, maxrt = 15, simult_conf = FALSE,
 z_absolute = FALSE, stop_on_error = TRUE, process_results = FALSE)
ddynaViTE(rt, response = "upper", th1, th2, a, v, t0 = 0, z = 0.5,
  d = 0, sz = 0, sv = 0, st0 = 0, tau = 1, w = 0.5, muvis = NULL,
  sigvis = 0, svis = 1, lambda = 0, s = 1, simult_conf = FALSE,
 precision = 6, z_absolute = FALSE, stop_on_error = TRUE,
  stop_on_zero = FALSE)
rdynaViTE(n, a, v, t0 = 0, z = 0.5, d = 0, sz = 0, sv = 0, st0 = 0,
  tau = 1, w = 0.5, muvis = NULL, sigvis = 0, svis = 1, lambda = 0,
  s = 1, delta = 0.01, maxrt = 15, simult_conf = FALSE,
  z_absolute = FALSE, stop_on_error = TRUE, process_results = FALSE)
```

#### **Arguments**

rt a vector of RTs. Or for convenience also a data.frame with columns rt and response.

response character vector, indicating the decision, i.e. which boundary was met first. Possible values are c("upper", "lower") (possibly abbreviated) and "upper" being the default. Alternatively, a numeric vector with values 1=lower and 2=upper or -1=lower and 1=upper, respectively. For convenience, response is con-

	verted via as.numeric also allowing factors. Ignored if the first argument is a ${\sf data.frame}$ .
th1	together with th2: scalars or numerical vectors giving the lower and upper bound of the interval of the confidence measure (see Details). Only values with th2>=th1 are accepted.
th2	(see th1)
a	threshold separation. Amount of information that is considered for a decision. Large values indicate a conservative decisional style. Typical range: $0.5 < a < 2$
V	drift rate of decision process. Average slope of the information accumulation process. The drift gives information about the speed and direction of the accumulation of information. Large (absolute) values of drift indicate a good performance. If received information supports the response linked to the upper threshold the sign will be positive and vice versa. Typical range: $-5 < v < 5$
t0	non-decision time or response time constant (in seconds). Lower bound for the duration of all non-decisional processes (encoding and response execution). Typical range: $0.1 < t0 < 0.5$ . Default is 0.
z	(by default relative) starting point of decision process. Indicator of an a priori bias in decision making. When the relative starting point z deviates from $0.5$ , the amount of information necessary for a decision differs between response alternatives. Default is $0.5$ (i.e., no bias).
d	differences in speed of response execution (in seconds). Positive values indicate that response execution is faster for responses linked to the upper threshold than for responses linked to the lower threshold. Typical range: $-0.1 < d < 0.1$ . Default is 0.
SZ	inter-trial-variability of starting point. Range of a uniform distribution with mean z describing the distribution of actual starting points from specific trials. Values different from 0 can predict fast errors (but can slow computation considerably). Typical range: $0 < sz < 0.2$ . Default is 0. (Given in relative range i.e. bounded by $2*\min(z, 1-z)$ )
sv	inter-trial-variability of drift rate of decision process. Standard deviation of a normal distribution with mean $\nu$ describing the distribution of actual drift rates from specific trials. Values different from 0 can predict slow errors. Typical range: $0 < s\nu < 2$ . Default is 0.
st0	inter-trial-variability of non-decisional components. Range of a uniform distribution with mean $t0 + st0/2$ describing the distribution of actual $t0$ values across trials. Accounts for response times below $t0$ . Reduces skew of predicted RT distributions. Values different from 0 can slow computation considerably. Typical range: $0 < st0 < 0.2$ . Default is 0.
tau	post-decisional accumulation time; the length of the time period after the decision was made until the confidence judgment is made. Range: tau>0. Default: tau=1.
W	weight put on the final state of the decision accumulator for confidence computation. 1-w is the weight on the visibility accumulator. Range: $0 < w < 1$ . Default: $w = 0.5$ .

muvis mean drift of visibility process. If NULL (default), muvis will be set to the absolute value of v.  sigvis the variability in drift rate of the visibility process (which varies independently from the drift rate in decision process). Range: sigvis>=0. Default: sigvis=0.  svis diffusion constant of visibility process. Range: svis>0. Default: svis=1.  lambda power for judgment time in the division of the confidence measure by the judgment time (Default: 0, i.e. no division which is the version of dynWEV propose by Hellmann et al., 2023)  s diffusion constant of decision process; standard deviation of the random noise of the diffusion process (i.e., within-trial variability), scales other parameters (see Note). Needs to be fixed to a constant in most applications. Default is 1. Not that the default used by Ratcliff and in other applications is often 0.1.  simult_conf logical. Whether in the experiment confidence was reported simultaneously with the decision. If that is the case decision and confidence judgment are assume to have happened subsequent before the response. Therefore tau is include in the response time. If the decision was reported before the confidence repor simul_conf should be FALSE.  precision numerical scalar value. Precision of calculation. Determines the step size of in
from the drift rate in decision process). Range: sigvis>=0. Default: sigvis=6  svis diffusion constant of visibility process. Range: svis>0. Default: svis=1.  lambda power for judgment time in the division of the confidence measure by the judgment time (Default: 0, i.e. no division which is the version of dynWEV propose by Hellmann et al., 2023)  s diffusion constant of decision process; standard deviation of the random noise of the diffusion process (i.e., within-trial variability), scales other parameters (see Note). Needs to be fixed to a constant in most applications. Default is 1. Not that the default used by Ratcliff and in other applications is often 0.1.  simult_conf logical. Whether in the experiment confidence was reported simultaneously with the decision. If that is the case decision and confidence judgment are assume to have happened subsequent before the response. Therefore tau is included in the response time. If the decision was reported before the confidence reports imul_conf should be FALSE.  precision numerical scalar value. Precision of calculation. Determines the step size of in
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·
tegration w.r.t. z and t0. Represents the number of decimals precisely compute on average. Default is 6.
z_absolute logical. Determines whether z is treated as absolute start point (TRUE) or relative (FALSE; default) to a.
stop_on_error Should the diffusion functions return 0 if the parameters values are outside the allowed range (= FALSE) or produce an error in this case (= TRUE).
Should the computation of densities stop as soon as a density value of 0 occur. This may save a lot of time if the function is used for a likelihood function Default: FALSE
n integer. The number of samples generated.
delta numeric. Discretization step size for simulations in the stochastic process
numeric. Maximum decision time returned. If the simulation of the stochasti process exceeds a decision time of maxrt, the response will be set to 0 and the maxrt will be returned as rt.
process_results
logical. Whether the output simulations should contain the final state of the decision (and visibility) process as additional column. Default is FALSE, meaning that no additional columns for the final process states are returned.

## **Details**

The function dWEV was renamed to ddynaViTE in version 0.1.0 of the package. It is still here for reasons of backwards compatibility. The function just calls the ddynaViTE function (and produces a deprecation warning).

The dynamical visibility, time, and evidence (dynaViTE) model and the weighted evidence and visibility model (dynWEV) are extensions of the 2DSD model for decision confidence (see d2DSD). It assumes that the decision follows a drift diffusion model with two additional assumptions to

account for confidence. First, there is a post-decisional period of further evidence accumulation tau. Second, another accumulation process accrues information about stimulus reliability (the visibility process) including also evidence about decision irrelevant features. See Hellmann et al. (2023) for more information. The measure for confidence is then a weighted sum of the final state of the decision process X and the visibility process V over a power-function of total accumulation time, i.e. for a decision time T (which is not the response time), the confidence variable is

$$conf = \frac{wX(T+\tau) + (1-w)V(T+\tau)}{(T+\tau)^{\lambda}}.$$

The dynWEV model is a special case of dynaViTE, with the parameter lambda=0.

All functions are fully vectorized across all parameters as well as the response to match the length or rt (i.e., the output is always of length equal to rt). This allows for trial wise parameters for each model parameter.

For convenience, the function allows that the first argument is a data.frame containing the information of the first and second argument in two columns (i.e., rt and response). Other columns (as well as passing response separately argument) will be ignored.

The functions dWEV and rWEV were renamed to ddynaViTE and rdynaViTE, respectively in version 0.1.0 of the package. They are still here for reasons of backwards compatibility. The functions just calls their counterpar ddynaViTE and rdynaViTE (and produce a deprecation warning).

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$$conf = \frac{wX(T+\tau) + (1-w)V(T+\tau)}{(T+\tau)^{\lambda}}.$$

The dynWEV model is a special case of dynaViTE, with the parameter lambda=0.

All functions are fully vectorized across all parameters as well as the response to match the length or rt (i.e., the output is always of length equal to rt). This allows for trial wise parameters for each model parameter.

For convenience, the function allows that the first argument is a data. frame containing the information of the first and second argument in two columns (i.e., rt and response). Other columns (as well as passing response separately argument) will be ignored.

#### Value

ddynaViTE gives the density/likelihood/probability of the diffusion process producing a decision of response at time rt and a confidence judgment corresponding to the interval [th1, th2]. The value will be a numeric vector of the same length as rt.

rdynaViTE returns a data. frame with three columns and n rows. Column names are rt (response time), response (-1 (lower) or 1 (upper), indicating which bound was hit), and conf (the value of the confidence measure; not discretized!).

The distribution parameters (as well as response, tau, th1 and th2, w and sig) are recycled to the length of the result. In other words, the functions are completely vectorized for all parameters and even the response boundary.

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rdynaViTE returns a data.frame with three columns and n rows. Column names are rt (response time), response (-1 (lower) or 1 (upper), indicating which bound was hit), and conf (the value of the confidence measure; not discretized!).

The distribution parameters (as well as response, tau, th1 and th2, w and sig) are recycled to the length of the result. In other words, the functions are completely vectorized for all parameters and even the response boundary.

#### Note

The parameterization of the non-decisional components, t0 and st0, differs from the parameterization sometimes used in the literature. In the present case t0 is the lower bound of the uniform distribution of length st0, but *not* its midpoint. The parameterization employed here is in line with the functions in the rtdists package.

The default diffusion constant s is 1 and not 0.1 as in most applications of Roger Ratcliff and others. Usually s is not specified as the other parameters: a, v, sv, muvis, sigvis, and svis respectively, may be scaled to produce the same distributions (as is done in the code).

The function code is basically an extension of the ddiffusion function from the package rtdists for the Ratcliff diffusion model.

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The function code is basically an extension of the ddiffusion function from the package rtdists for the Ratcliff diffusion model.

#### Author(s)

Sebastian Hellmann

#### References

Hellmann, S., Zehetleitner, M., & Rausch, M. (2023). Simultaneous modeling of choice, confidence and response time in visual perception. *Psychological Review* 2023 Mar 13. doi: 10.1037/rev0000411. Epub ahead of print. PMID: 36913292.

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#### **Examples**

```
# Plot rt distribution ignoring confidence
curve(ddynaViTE(x, "upper", -Inf, Inf, tau=1, a=2, v=0.4, sz=0.2, sv=0.9), xlim=c(0, 2), lty=2)
curve(ddynaViTE(x, "lower", -Inf, Inf,tau=1, a=2, v=0.4, sz=0.2, sv=0.9), col="red", lty=2, add=TRUE)
curve(ddynaViTE(x, "upper", -Inf, Inf, tau=1, a=2, v=0.4),add=TRUE)
curve(ddynaViTE(x, "lower", -Inf, Inf, tau=1, a=2, v=0.4), col="red", add=TRUE)
# Generate a random sample
df1 <- rdynaViTE(5000, a=2,v=0.5,t0=0,z=0.5,d=0,sz=0,sv=0, st0=0, tau=1, s=1, w=0.9)
# Same RT and response distribution but different confidence distribution
df2 <- rdynaViTE(5000, a=2,v=0.5,t0=0,z=0.5,d=0,sz=0,sv=0, st0=0, tau=1, s=1, w=0.1)
head(df1)
# Scaling diffusion parameters leads do same density values
ddynaViTE(df1[1:5,], th1=-Inf, th2=Inf, a=2, v=.5)[1:5]
ddynaViTE(df1[1:5,], th1=-Inf, th2=Inf, a=2*s, v=.5*s, s=2)[1:5]
# Diffusion constant also scales confidence parameters
ddynaViTE(df1[1:5,], th1=0.2, th2=1, a=2, v=.5, sv=0.2, w=0.5, sigvis = 0.2, svis = 1)[1:5]
s <- 2
ddynaViTE(df1[1:5,], th1=0.2*s, th2=1*s, a=2*s, v=.5*s, s=2,
     sv=0.2*s, w=0.5, sigvis=0.2*s, svis=1*s)[1:5]
two_samples <- rbind(cbind(df1, w="high"),</pre>
                     cbind(df2, w="low"))
# no difference in RT distributions
boxplot(rt~w+response, data=two_samples)
# but different confidence distributions
boxplot(conf~w+response, data=two_samples)
if (requireNamespace("ggplot2", quietly = TRUE)) {
  require(ggplot2)
 ggplot(two_samples, aes(x=rt, y=conf))+
    stat_density_2d(aes(fill = after_stat(density)), geom = "raster", contour = FALSE) +
    x\lim(c(0, 2)) + y\lim(c(-1.5, 4)) +
    facet_grid(cols=vars(w), rows=vars(response), labeller = "label_both")
}
# Restricting to specific confidence region
df1 <- df1[df1$conf >0 & df1$conf <1,]</pre>
ddynaViTE(df1[1:5,], th1=0, th2=1, a=2, v=0.5)[1:5]
# If lower confidence threshold is higher than the upper, the function throws an error,
# except when stop_on_error is FALSE
ddynaViTE(df1[1:5,], th1=1, th2=0, a=2, v=0.5, stop_on_error = FALSE)
# Plot rt distribution ignoring confidence
curve(ddynaViTE(x, "upper", -Inf, Inf, tau=1, a=2, v=0.4,
                    sz=0.2, sv=0.9),
     xlim=c(0, 2), lty=2)
curve(ddynaViTE(x, "lower", -Inf, Inf,tau=1, a=2, v=0.4,
```

sz=0.2, sv=0.9),

```
col="red", lty=2, add=TRUE)
curve(ddynaViTE(x, "upper", -Inf, Inf, tau=1, a=2, v=0.4),add=TRUE)
curve(ddynaViTE(x, "lower", -Inf, Inf, tau=1, a=2, v=0.4), col="red", add=TRUE)
# Generate a random sample
df1 <- rdynaViTE(5000, a=2,v=0.5,t0=0,z=0.5,d=0,sz=0,sv=0, st0=0, tau=1, s=1, w=0.9)
# Same RT and response distribution but different confidence distribution
df2 <- rdynaViTE(5000, a=2,v=0.5,t0=0,z=0.5,d=0,sz=0,sv=0, st0=0, tau=1, s=1, w=0.1)
head(df1)
# Scaling diffusion parameters leads do same density values
ddynaViTE(df1[1:5,], th1=-Inf, th2=Inf, a=2, v=.5)[1:5]
s <- 2
ddynaViTE(df1[1:5,], th1=-Inf, th2=Inf, a=2*s, v=.5*s, s=2)[1:5]
# Diffusion constant also scales confidence parameters
ddynaViTE(df1[1:5,], th1=0.2, th2=1, a=2, v=.5, sv=0.2, w=0.5, sigvis = 0.2, svis = 1)[1:5]
s <- 2
ddynaViTE(df1[1:5,], th1=0.2*s, th2=1*s, a=2*s, v=.5*s, s=2,
     sv=0.2*s, w=0.5, sigvis=0.2*s, svis=1*s)[1:5]
two_samples <- rbind(cbind(df1, w="high"),</pre>
                     cbind(df2, w="low"))
# no difference in RT distributions
boxplot(rt~w+response, data=two_samples)
# but different confidence distributions
boxplot(conf~w+response, data=two_samples)
if (requireNamespace("ggplot2", quietly = TRUE)) {
 require(ggplot2)
 ggplot(two_samples, aes(x=rt, y=conf))+
   stat_density_2d(aes(fill = after_stat(density)), geom = "raster", contour = FALSE) +
   x\lim(c(0, 2)) + y\lim(c(-1.5, 4)) +
    facet_grid(cols=vars(w), rows=vars(response), labeller = "label_both")
}
# Restricting to specific confidence region
df1 <- df1[df1$conf >0 & df1$conf <1,]</pre>
ddynaViTE(df1[1:5,], th1=0, th2=1, a=2, v=0.5)[1:5]
# If lower confidence threshold is higher than the upper, the function throws an error,
# except when stop_on_error is FALSE
ddynaViTE(df1[1:5,], th1=1, th2=0, a=2, v=0.5, stop_on_error = FALSE)
```

fitRTConf

Function for fitting sequential sampling confidence models

#### **Description**

Fits the parameters of different models of response time and confidence, including the 2DSD model (Pleskac & Busemeyer, 2010), dynWEV, DDConf, and various flavors of race models (Hellmann

et al., 2023). Which model to fit is specified by the argument model. Only a ML method is implemented. See ddynaViTE, d2DSD, and dRM for more information about the parameters and Details for not-fitted parameters.

## Usage

```
fitRTConf(data, model = "dynWEV", fixed = list(sym_thetas = FALSE),
  init_grid = NULL, grid_search = TRUE, data_names = list(),
  nRatings = NULL, restr_tau = Inf, precision = 6, logging = FALSE,
  opts = list(), optim_method = "bobyqa", useparallel = FALSE,
  n.cores = NULL, ...)
```

#### **Arguments**

data

a data. frame where each row is one trial, containing following variables (column names can be changed by passing additional arguments of the form condition="contrast"):

- condition (not necessary; for different levels of stimulus quality, will be transformed to a factor).
- rating (discrete confidence judgments, should be given as integer vector; otherwise will be transformed to integer),
- rt (giving the reaction times for the decision task),
- either 2 of the following (see details for more information about the accepted formats):
  - stimulus (encoding the stimulus category in a binary choice task),
  - response (encoding the decision response),
  - correct (encoding whether the decision was correct; values in 0, 1)
- sbj or participant (optional; giving the subject ID; only relevant if logging = TRUE; if unique the ID is used in saved files with interim results and logging messages; if non-unique or missing and logging =TRUE, 999 will be used then)

model

character scalar. One of "dynWEV", "2DSD", "IRM", "PCRM", "IRMt", "PCRMt", or "DDConf" for the model to be fit.

fixed

list. List with parameter-value pairs for parameters that should not be fitted. See Details.

init\_grid

data.frame or NULL. Grid for the initial parameter search. Each row is one parameter constellation. See details for more information. If NULL a default grid will be used.

grid\_search

logical. If FALSE, the grid search before the optimization algorithm is omitted. The fitting is then started with a mean parameter set from the default grid (if init\_grid=NULL) or directly with the rows from init\_grid, if not NULL. (Default: TRUE)

data\_names

named list (e.g. c(rating="confidence")). Alternative possibility of giving other column names for the variables in the data. By default column names are identical to the ones given in the data argument description.

nRatings	integer. Number of rating categories. If NULL, the maximum of rating and length(unique(rating)) is used. This argument is especially important for data sets where not the whole range of rating categories is realized. If given, ratings has to be given as factor or integer.
restr_tau	numerical or Inf or "simult_conf". For 2DSD and dynWEV only. Upper bound for tau. Fits will be in the interval (0,restr_tau). If FALSE tau will be unbound. For "simult_conf", see the documentation of d2DSD and ddynaViTE
precision	numeric. Precision of calculation for the density functions (see ddynaViTE and dPCRM for more information).
logging	logical. If TRUE, a folder 'autosave/fit <b>model</b> ' is created and messages about the process are printed in a logging file and to console (depending on OS). Additionally intermediate results are saved in a .RData file with the participant ID in the name.
opts	list. A list for more control options in the optimization routines (depending on the optim_method). See details for more information.
optim_method	character. Determines which optimization function is used for the parameter estimation. Either "bobyqa" (default), "L-BFGS-B" or "Nelder-Mead". "bobyqa" uses a box-constrained optimization with quadratic interpolation. (See bobyqa for more information.) The first two use a box-constraint optimization. For Nelder-Mead a transfinite function rescaling is used (i.e. the constrained arguments are suitably transformed to the whole real line).
useparallel	logical. If TRUE the grid search in the beginning is done with a parallel back-end, using the parallel package.
n.cores	integer or NULL. Number of cores used for parallelization. If NULL (default) the number of available cores -1 is used.
• • •	Possibility of giving alternative variable names in data frame (in the form condition = "SOA", or response="pressedKey").

## Details

The fitting involves a first grid search through computation of the likelihood on an initial grid with possible sets of parameters to start the optimization routine. Then the best nAttempts parameter sets are chosen for an optimization, which is done with an algorithm, depending on the argument optim-method. The Nelder-Mead algorithm uses the R function optim. The optimization routine is restarted nRestarts times with the starting parameter set equal to the best parameters from the previous routine.

stimulus, response and correct. Two of these columns must be given in data. If all three are given, correct will have no effect (and will be not checked!). stimulus can always be given in numerical format with values -1 and 1. response can always be given as a character vector with "lower" and "upper" as values. Correct must always be given as a 0-1-vector. If the stimulus column is given together with a response column and they both do not match the above format, they need to have the same values/levels (if factor). In the case that only stimulus/response is given in any other format together with correct, the unique values will be sorted increasingly and the first value will be encoded as "lower"/-1 and the second as "upper"/+1.

**fixed**. Parameters that should not be fitted but kept constant. These will be dropped from the initial grid search but will be present in the output, to keep all parameters for prediction in the result. Includes the possibility for symmetric confidence thresholds for both alternative (sym\_thetas=logical).

Other examples are z = .5, sv=0, st0=0, sz=0. For race models, the possibility of setting a='b' (or vice versa) leads to identical upper bounds on the decision processes, which is the equivalence for z=.5 in a diffusion process.

**Parameters not fitted**. The models get developed continuously and not all changes are adopted in the fitting function instantly. Following parameters are currently not included in the fitting routine:

• in race models: sza, szb, smu1, and smu2

init\_grid. Each row should be one parameter set to check. The column names should include the parameters of the desired model, which are the following for 2DSD: a, vmin and vmax (will be equidistantly spanned across conditions), sv, z (as the relative starting point between 0 and a), sz (also in relative terms), t0, st0, theta0 (minimal threshold), thetamax (maximal threshold; the others will be equidistantly spanned symmetrically for both decisions), and tau. For dynWEV, additionally w, svis, and sigvis are required. For the race models the parameters are: vmin, vmax (will be equidistantly spanned across conditions), a and b (decision thresholds), t0, st0, theta0 (minimal threshold), thetamax (maximal threshold; the others will be equidistantly spanned symmetrically for both decisions), and for time-dependent confidence race models additionally wrt and wint (as weights compared to wx=1).

**opts**. A list with numerical values. Possible options are listed below (together with the optimization method they are used for).

- nAttempts (all) number of best performing initial parameter sets used for optimization; default 5, if grid\_search is TRUE. If grid\_search is FALSE and init\_grid is NULL, then nAttempts will be set to 1 (and any input will be ignored). If grid\_search is FALSE and init\_grid is not NULL, the rows of init\_grid will be used from top to bottom (since no initial grid search is done) with not more than nAttempts rows used.
- nRestarts (all) number of successive optim routines for each of the starting parameter sets; default 5,
- maxfun ('bobyqa') maximum number of function evaluations; default: 5000,
- maxit ('Nelder-Mead' and 'L-BFGS-B') maximum iterations; default: 2000,
- reltol ('Nelder-Mead') relative tolerance; default: 1e-6),
- factr ('L-BFGS-B') tolerance in terms of reduction factor of the objective, default: 1e-10)

#### Value

Gives a one-row data frame with columns for the different parameters as fitted result as well as additional information about the fit (negLogLik (for final parameters), k (number of parameters), N (number of data rows), BIC, AICc and AIC) and the column fixed, which includes all information about fixed and not fitted parameters.

### Author(s)

Sebastian Hellmann.

#### References

Hellmann, S., Zehetleitner, M., & Rausch, M. (2023). Simultaneous modeling of choice, confidence and response time in visual perception. *Psychological Review* 2023 Mar 13. doi: 10.1037/rev0000411. Epub ahead of print. PMID: 36913292.

```
https://nashjc.wordpress.com/2016/11/10/why-optim-is-out-of-date/https://www.damtp.cam.ac.uk/user/na/NA_papers/NA2009_06.pdf
```

## **Examples**

```
# We use one of the implemented models, "dynWEV"
# 1. Generate data
# data with positive drift (stimulus = "upper")
data <- rdynaViTE(20, a=2,v=0.5,t0=0.2,z=0.5, sz=0.1,sv=0.1, st0=0, tau=4, s=1, w=0.3)
data$stimulus <- "upper"
# data with negtive drift (stimulus = "lower") but same intensity
data2 <- rdynaViTE(100, a=2,v=-0.5,t0=0.2,z=0.5,sz=0.1,sv=0.1, st0=0, tau=4, s=1, w=0.3)
data2$stimulus <- "lower"
data <- rbind(data, data2)
# Transfer response column and add dummy condition column
data$response <- ifelse(data$response==1, "upper", "lower")</pre>
data$condition <- 1
# Take some confidence thresholds for discrete ratings
threshs < c(-Inf, 1, 2, Inf)
data$rating <- as.numeric(cut(data$conf, breaks = threshs, include.lowest = TRUE))</pre>
head(data)
# 2. Use fitting function
# Fitting the model with these opts results in a pretty bad fit
# (especially because of omitting the grid_search)
   fitRTConf(data, "dynWEV", fixed=list(sym_thetas=TRUE, z=0.5, st0=0),
            grid_search = FALSE, logging=FALSE,
            opts = list(nAttempts=1, nRestarts=2, maxfun=2000))
```

fitRTConfModels

Function for fitting several sequential sampling confidence models in parallel

## Description

This function is a wrapper of the function fitConfModel (see there for more information). It calls the function for every possible combination of model and participant in model and data respectively. Also, see ddynaViTE, d2DSD, dDDConf, and dRM for more information about the parameters.

#### Usage

```
fitRTConfModels(data, models = c("dynaViTE", "2DSD", "PCRMt"),
 nRatings = NULL, fixed = list(sym_thetas = FALSE), restr_tau = Inf,
 grid_search = TRUE, opts = list(), optim_method = "bobyqa",
 logging = FALSE, precision = 6, parallel = TRUE, n.cores = NULL, ...)
```

### **Arguments**

data

a data. frame where each row is one trial, containing following variables (column names can be changed by passing additional arguments of the form condition="contrast"):

- condition (not necessary; for different levels of stimulus quality, will be transformed to a factor),
- rating (discrete confidence judgments, should be given as integer vector; otherwise will be transformed to integer),
- rt (giving the reaction times for the decision task),
- either 2 of the following (see details for more information about the accepted formats):
  - stimulus (encoding the stimulus category in a binary choice task),
  - response (encoding the decision response),
  - correct (encoding whether the decision was correct; values in 0, 1)
- sbj (giving the subject ID; the models given in the second argument are fitted for each subject individually. (Furthermore, if logging = TRUE, the ID is used in files saved with interim results and logging messages.))

models

character vector with following possible elements "dynWEV", "2DSD", "IRM", "PCRM", "IRMt", and "PCRMt" for the models to be fit.

nRatings

integer. Number of rating categories. If NULL, the maximum of rating and length(unique(rating)) is used. This argument is especially important for data sets where not the whole range of rating categories is realized. If given, ratings has to be given as factor or integer.

fixed

list. List with parameter value pairs for parameters that should not be fitted. (see Details).

restr\_tau

numerical or Inf or "simult\_conf". Used for 2DSD and dynWEV only. Upper bound for tau. Fits will be in the interval (0,restr\_tau). If FALSE tau will be unbound. For "simult\_conf", see the documentation of d2DSD and ddynaViTE

grid\_search

logical. If FALSE, the grid search before the optimization algorithm is omitted. The fitting is then started with a mean parameter set from the default grid. (Default: TRUE)

opts

list. A list for more control options in the optimization routines (depending on the optim\_method). See details for more information.

optim\_method

character. Determines which optimization function is used for the parameter estimation. Either "bobyga" (default), "L-BFGS-B" or "Nelder-Mead". "bobyga" uses a box-constrained optimization with quadratic interpolation. (See bobyga for more information.) The first two use a box-constraint optimization. For Nelder-Mead a transfinite function rescaling is used (i.e. the constrained arguments are suitably transformed to the whole real line).

logging logical. If TRUE, a folder 'autosave/fitmodel' is created and messages about the process are printed in a logging file and to console (depending on OS). Addi-

tionally intermediate results are saved in a .RData file with the participant ID in

the name.

precision numerical numeric. Precision of calculation for the density functions (see ddynaViTE

and dPCRM for more information).

parallel "models", "single", "both" or FALSE. If FALSE no parallelization is used in the

fitting process. If "models" the fitting process is parallelized over participants and models (i.e. over the calls for fitting functions). If "single" parallelization is used within the fitting processes (over initial grid search and optimization processes for different start points, but see fitRTConf). If "both", parallelization is done hierarchical. For small number of models and participants "single" or

"both" is preferable. Otherwise, you may use "models".

n.cores integer vector or NULL. If parallel is "models" or "single", a single integer for

the number of cores used for parallelization is required. If parallel is "both", two values are required. The first for the number of parallel model-participant combinations and the second for the parallel processes within the fitting procedures (this may be specified to match the nAttemps-Value in the opts argument. If NULL (default) the number of available cores -1 is used. If NULL and parallel

is "both", the cores will be used for model-participant-parallelization, only.

Possibility of giving alternative variable names in data frame (in the form condition

= "SOA", or response="pressedKey").

#### **Details**

The fitting involves a first grid search through an initial grid. Then the best nAttempts parameter sets are chosen for an optimization, which is done with an algorithm, depending on the argument optim-method. The Nelder-Mead algorithm uses the R function optim. The optimization routine is restarted nRestarts times with the starting parameter set equal to the best parameters from the previous routine.

stimulus, response and correct. Two of these columns must be given in data. If all three are given, correct will have no effect (and will be not checked!). stimulus can always be given in numerical format with values -1 and 1. response can always be given as a character vector with "lower" and "upper" as values. Correct must always be given as a 0-1-vector. If stimulus is given together with response and they both do not match the above format, they need to have the same values/levels (if factor). In the case that only stimulus/response is given in any other format together with correct, the unique values will be sorted increasingly and the first value will be encoded as "lower"/-1 and the second as "upper"/+1.

**fixed**. Parameters that should not be fitted but kept constant. These will be dropped from the initial grid search but will be present in the output, to keep all parameters for prediction in the result. Includes the possibility for symmetric confidence thresholds for both alternative (sym\_thetas=logical). Other examples are z = .5, sv=0, st0=0, sz=0. For race models, the possibility of setting a='b' (or vice versa) leads to identical upper bounds on the decision processes, which is the equivalence for z=.5 in a diffusion process

**opts**. A list with numerical values. Possible options are listed below (together with the optimization method they are used for).

nAttempts (all) number of best performing initial parameter sets used for optimization; default 5

- nRestarts (all) number of successive optim routines for each of the starting parameter sets;
   default 5
- maxfun ('bobyqa') maximum number of function evaluations; default: 5000,
- maxit ('Nelder-Mead' and 'L-BFGS-B') maximum iterations; default: 2000,
- reltol ('Nelder-Mead') relative tolerance; default: 1e-6),
- factr ('L-BFGS-B') tolerance in terms of reduction factor of the objective, default: 1e-10)

#### Value

Gives data frame with rows for each model-participant combination and columns for the different parameters as fitted result as well as additional information about the fit (negLogLik (for final parameters), k (number of parameters), N (number of data rows), BIC, AICc and AIC)

#### Author(s)

Sebastian Hellmann, <sebastian.hellmann@ku.de>

#### References

Hellmann, S., Zehetleitner, M., & Rausch, M. (2023). Simultaneous modeling of choice, confidence and response time in visual perception. *Psychological Review* 2023 Mar 13. doi: 10.1037/rev0000411. Epub ahead of print. PMID: 36913292.

#### **Examples**

```
# 1. Generate data from two artificial participants
# Get random drift direction (i.e. stimulus category) and
# stimulus discriminability (two steps: hard, easy)
stimulus \leftarrow sample(c(-1, 1), 400, replace=TRUE)
discriminability <- sample(c(1, 2), 400, replace=TRUE)
# generate data for participant 1
data <- rdynaViTE(400, a=2, v=stimulus*discriminability*0.5,</pre>
             t0=0.2, z=0.5, sz=0.1, sv=0.1, st0=0, tau=4, s=1, w=0.3)
# discretize confidence ratings (only 2 steps: unsure vs. sure)
data$rating <- as.numeric(cut(data$conf, breaks = c(-Inf, 1, Inf), include.lowest = TRUE))</pre>
data$participant = 1
data$stimulus <- stimulus
data$discriminability <- discriminability
# generate data for participant 2
data2 <- rdynaViTE(400, a=2.5, v=stimulus*discriminability*0.7,
             t0=0.1, z=0.7, sz=0, sv=0.2, st0=0, tau=2, s=1, w=0.5)
data2$rating <- as.numeric(cut(data$conf, breaks = c(-Inf, 0.3, Inf), include.lowest = TRUE))
data2participant = 2
data2$stimulus <- stimulus
data2$discriminability <- discriminability</pre>
# bind data from participants
```

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LogLikRM

Log-Likelihood functions for the independent and partially anticorrelated race models of confidence

## **Description**

Computes the Log-likelihood for given data and parameters in the IRM and PCRM with or without time-scaled confidence measure. It is a wrapped version of the respective densities dIRM and dPCRM, where one can find more information about the parameters. It restricts the rates of accumulation to be the negative of each other, though (a common assumption in perceptual decision tasks). The function is mainly used inside fitRTConf for race models but exported for individual usage in other contexts.

## Usage

```
LogLikRM(data, paramDf, model = "IRM", time_scaled = FALSE,
    precision = 6, data_names = list(), ...)
```

#### **Arguments**

data

a dataframe where each row is one trial. Containing following variables:

- condition (not necessary; convertible to integer (e.g. factor); for different levels of stimulus quality),
- rating (convertible to integer (e.g. factor); discrete confidence judgments),
- rt (numeric; giving reaction times for decision task),
- stimulus (values at least convertible to c(1,2), i.e. integer or factor; stimulus category (index of accumulator with higher drift))
- response (values at least convertible to c(1,2); direction of decision; (index of accumulator reaching the boundary first))

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paramDf a list or data frame with one row. Column names should match the names of RaceModels parameter names (only mu1 and mu2 are not used in this context but replaced by the parameter v). For different stimulus quality/mean drift rates, names should be v1, v2, v3,.... Different s parameters are possible with s1, s2, s3,... with equally many steps as for drift rates. Additionally, the confidence thresholds should be given by names with thetaUpper1, thetaUpper2,..., thetaLower1,... or, for symmetric thresholds only by theta1, theta2,.... (see Details for the correspondence to the data) mode1 character scalar. One of "IRM" or "PCRM". ("IRMt" and "PCRMt" will also be accepted. In that case, time\_scaled is set to TRUE.) time\_scaled logical. Whether the confidence measure should be scaled by 1/sqrt(rt). Default: TRUE. numerical scalar. Precision of calculation for integration over t0. precision data\_names list. Possibility of giving alternative column names for the variables in the data. By default column names are identical to the ones given in the data argument description. Another possibility of giving alternative variable names in data frame (in the form condition = "SOA").

#### **Details**

Note, that the requirements on the format of the columns for the likelihood functions are much stricter, than in fitRTConf. This is because the function is very frequently called in the optimization routines of the fitting process and the preprocessing steps are therefore included in the other function.

rating, condition. If integer, values should range from 1 to number of possible ratings/conditions. If factor, the number of levels should be equal to number of possible ratings/conditions. This should be consistent with the parameter vector. The confidence thresholds should be named as thetaUpper1, thetaLower1,.... (or theta1,... for symmetric thresholds), with the number of ratings -1 and the mean drift rates (and possibly the standard deviation in drift rates) should be denoted as v1, v2,... (and s1, s2,...) with the number equal to the number of conditions. If only one condition is used v will be accepted as well as v1.

**stimulus, response**. stimulus and response should always be given in numerical format with values 1 and 2. Stimulus determines which of two accumulators has positive drift. The other has negative drift with the same absolute value. Response gives the index of the accumulator that reaches the boundary first.

#### Value

Numeric scalar. The summed Log-likelihood of the data given the parameters in the respective model. If one or more row-wise probabilities is <=0, the function returns -1e+12.

#### Author(s)

Sebastian Hellmann.

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#### **Examples**

```
# 1. Generate data from an artificial participants
# Get random index for accumulator with positive
# drift (i.e. stimulus category) and
# stimulus discriminability (two steps: hard, easy)
stimulus <- sample(c(1, 2), 200, replace=TRUE)</pre>
discriminability <- sample(c(1, 2), 200, replace=TRUE)</pre>
# generate data for participant 1
data <- rPCRM(200, mu1=ifelse(stimulus==1, 1, -1)*discriminability*0.5,
              mu2=ifelse(stimulus==1, -1, 1)*discriminability*0.5,
             a=2, b=1.8, t0=0.2, st0=0, wx=0.7, wint=0.3, wrt=0)
# discretize confidence ratings (only 2 steps: unsure vs. sure)
data$rating <- as.numeric(cut(data$conf, breaks = c(0, 3, Inf), include.lowest = TRUE))</pre>
data$participant = 1
data$stimulus <- stimulus
data$discriminability <- discriminability
data <- data[data$response!=0, ] # drop not finished decision processes
data <- data[,-c(3,4)] # drop xl and conf measure (unobservable variable)</pre>
head(data)
# 2. Define some parameter set in a data.frame
paramDf \leftarrow data.frame(a=2,b=2, v1=0.5, v2=1, t0=0.1,st0=0,
                      wx=0.6, wint=0.2, wrt=0.2,
                      theta1=4)
# 3. Compute log likelihood for parameter and data
LogLikRM(data, paramDf, model="PCRMt", condition="discriminability")
# same result
LogLikRM(data, paramDf, model="PCRM", time_scaled=TRUE,condition="discriminability")
# different
LogLikRM(data, paramDf, model="PCRM", condition="discriminability")
# same parameters used for IRM model
LogLikRM(data, paramDf, model="IRMt", condition="discriminability")
```

LogLikWEV

Log-Likelihood functions for the dynWEV and 2DSD models of confidence

## **Description**

Computes the Log-likelihood for given data and parameters in the dynWEV model (Hellmann et al., 2023) and the 2DSD model (Pleskac & Busemeyer, 2010). It is a wrapped version of the respective densities ddynaViTE and d2DSD, where one can find more information about the parameters (z is always given relatively, in the likelihood). The function is mainly used in fitRTConf but exported for individual usage in other contexts.

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#### Usage

```
LogLikWEV(data, paramDf, model = "dynaViTE", simult_conf = FALSE,
 precision = 6, stop_on_error = TRUE, data_names = list(), ...)
```

#### **Arguments**

data

a dataframe where each row is one trial. Containing following variables:

- condition (not necessary; convertible to integer (e.g. factor); for different levels of stimulus quality),
- rating (convertible to integer (e.g. factor); discrete confidence judgments),
- rt (numeric; giving reaction times for decision task),
- stimulus (values at least convertible to c(-1,1); stimulus category (direction of evidence accumulation))
- response (characters in "upper", "lower" (or convertible to); direction of decision; correct answers are "lower" for stimulus=-1; and "upper" for stimulus=1),

list or data.frame with one row. Names should match the names of dynaViTE and 2DSD model specific parameter names. For different stimulus quality/mean drift rates, names should be v1, v2, v3,.... Different sv and/or s parameters are possible with sv1, sv2, sv3... (s1, s2, s3,... respectively) with equally many steps as for drift rates. Additionally, the confidence thresholds should be given by names with thetaUpper1, thetaUpper2,..., thetaLower1,... or, for symmetric thresholds only by theta1, theta2,... (see Details for the correspondence to the data)

character scalar. One of "dynWEV" or "2DSD" for the model to fit. mode1

logical. Whether in the experiment confidence was reported simultaneously with simult\_conf

the decision, as then decision and confidence judgment are assumed to have happened subsequent before response and computations are different, when there is

an observable interjudgment time (then simult\_conf should be FALSE).

numerical scalar. Precision of calculation for integration over z and t0. precision

logical. If TRUE an error in the function will be returned in case of invalid stop\_on\_error

parameters. Otherwise, the output will be 0 without error.

data\_names list. Possibility of giving alternative column names for the variables in the data.

By default column names are identical to the ones given in the data argument

description.

Possibility of giving alternative variable names in data frame (in the form condition

= "SOA").

#### **Details**

Note, that the requirements on the format of the columns for the likelihood functions are much stricter, than in fitRTConf. This is because the function is very frequently calls in the optimization routines of the fitting process and the preprocessing steps are therefore included in that function.

rating, condition. If integer, values should range from 1 to number of possible ratings/conditions. If a factor, the number of levels should be equal to number of possible ratings/conditions. This

paramDf

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should be consistent with the parameter vector. The confidence thresholds should be named as thetaUpper1, thetaLower1,.... (or theta1,... for symmetric thresholds), with the number of ratings -1 and the mean drift rates (and possibly the standard deviation in drift rates) should be denoted as v1, v2,... (and sv1, sv2,.../s1, s2, ...) with the number equal to the number of conditions. If only one condition is used v will be accepted as well as v1.

**stimulus, response.** stimulus should always be given in numerical format with values -1 and 1. response should always be given as a character vector with "lower" and "upper" as values. This corresponds to the situation of Ratcliff's diffusion model (Ratcliff, 1978), where stimulus is the sign of the mean drift direction and the response is the "upper" or "lower" boundary that is first hit by the evidence accumulation. A correct decision is therefore "lower", if stimulus is -1, and "upper", if stimulus is 1.

#### Value

Numeric scalar. The summed Log-likelihood of the data given the parameters in the respective model. If one or more row-wise probabilities is <=0, the function returns -1e+12.

## Author(s)

Sebastian Hellmann.

#### References

Hellmann, S., Zehetleitner, M., & Rausch, M. (2023). Simultaneous modeling of choice, confidence and response time in visual perception. *Psychological Review* 2023 Mar 13. doi: 10.1037/rev0000411. Epub ahead of print. PMID: 36913292.

## **Examples**

```
# 1. Generate data from an artificial participants
# Get random drift direction (i.e. stimulus category) and
# stimulus discriminability (two steps: hard, easy)
stimulus <- sample(c(-1, 1), 200, replace=TRUE)
discriminability <- sample(c(1, 2), 200, replace=TRUE)
# generate data for participant 1
data <- rdynaViTE(200, a=2,v=stimulus*discriminability*0.5,</pre>
             t0=0.2, z=0.5, sz=0.1, sv=0.1, st0=0, tau=4, s=1, w=0.3
# discretize confidence ratings (only 2 steps: unsure vs. sure)
data$rating <- as.numeric(cut(data$conf, breaks = c(-Inf, 1, Inf), include.lowest = TRUE))</pre>
data$participant = 1
data$stimulus <- stimulus
data$discriminability <- discriminability
data <- data[data$response!=0, ] # drop not finished decision processes
data <- data[,-3] # drop conf measure (unobservable variable)</pre>
head(data)
# 2. Define some parameter set in a data.frame
paramDf \leftarrow data.frame(a=2.5,v1=0.5, v2=1, t0=0.1,z=0.7,
                      sz=0,sv=0.2, st0=0, tau=3, w=0.3,
                      theta1=0.8, svis=0.5, sigvis=0.8)
```

32 PDFtoQuantiles

**PDFtoQuantiles** 

Get Quantiles from vectors of PDF or CDF values

#### **Description**

CDFtoQuantiles computes quantiles for a given CDF. PDFtoQuantiles computes the quantiles for given PDF values within groups of other variables, if available.

#### Usage

```
PDFtoQuantiles(pdf_df, p = c(0.1, 0.3, 0.5, 0.7, 0.9), agg_over = NULL,
    scaled = FALSE)

CDFtoQuantiles(cdf, x = NULL, p)
```

#### **Arguments**

pdf\_df

dataframe. Should have at least two columns:

- rt (for reaction times) or x for the support values of the pdf
- dens or pdf for the pdf values
- All other columns will be used as grouping factors, for which separate quantiles will be returned.

р

numeric vector. Probabilities for returned quantiles. Default: c(.1, .3, .5, .7, .9).

agg\_over

character. Names of columns in pdf\_df to aggregate over (using the mean of densities, which is valid only, if groups occur with equal probabilities) before computing the quantiles.

scaled

logical. Indicating whether the pdf values are from a proper probability distribution. Non-scaled pdfs will scaled to 1. If scaled is TRUE, this may cause problems with high probabilities. In any case we strongly recommend to cover the most probability mass with the values in the support vector.

cdf

numeric. A increasing vector of the same length as x giving the CDF for respective x-Values. Dataframe inputs are accepted. If a column x is available there, this will be used as support values.

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x numeric. A increasing vector of same length as cdf. Can also be specified as column of cdf.

#### **Details**

For a reasonable accuracy the number of steps in the support column (rt/x) should be high, i.e. the distance between values small. We recommend, to ensure that the support vector in the input to be equidistant, i.e. the difference between consecutive support values should be constant, though this is not required. If both column names x and rt are present in pdf\_df, rt will be preferred. Attention should be given to the columns of pdf\_df other than rt/x and dens/pdf.

The column for the pdf may be scaled to integrate to 1 but do not have to.

### Quantile computation in the dynConfiR package:

As argument pdf\_df, the outputs of predictRT and predictRTModels from the dynConfiR package can be used. In the context of confidence models grouping factors often used are conditions, correct/incorrect answers and confidence ratings.

#### Value

PDFtoQuantiles returns a tibble with columns p and q indicating probabilities and respective quantiles. Furthermore, the output has grouping columns identical to the additional columns in the input (without rt/x, dens and densscaled), but without the ones in the agg\_over argument. CDFtoQuantiles returns only a data.frame with columns p and q.

#### Author(s)

Sebastian Hellmann.

#### **Examples**

```
## Demonstrate PDFtoQuantiles
pred <- expand.grid(model = c("dynWEV", "PCRMt"),</pre>
                     rt = seq(0, 15, length.out=1200),
                     condition = c(1,2,3),
                     rating = c(1,2))
pred$dens <- dchisq(pred$rt, 3) # pdf may also be used as column name</pre>
head(pred)
res <- PDFtoQuantiles(pred, p=c(0.3, 0.5, 0.7))
head(res)
nrow(res) #= 3(quantiles)*2(models)*3(conditions)*2(rating)
# Compare to true quantiles of Chi-square distribution
qchisq(p=c(0.3, 0.5, 0.7), 3)
res$q[1:3]
res2 <- PDFtoQuantiles(pred, p=c(0.3, 0.5, 0.7), agg_over = "model")
nrow(res2) #=18 because res aggregated over models
  pred$pdf <- dchisq(pred$rt, 3)</pre>
  head(pred)
```

34 predictDDConf

```
# following call throws a warning, because both columns pdf and dens are present
PDFtoQuantiles(pred, p=c(0.3, 0.5, 0.7), agg_over = "model")

pred2 <- data.frame(rt=seq(0, 7, length.out=100))
pred2$dens <- dchisq(pred2$rt, 5)

# following call throws a warning, because density is assumed to be scaled (scaled=TRUE), i.e.
# integrate to 1, but the .95 quantile is not reached in the rt column
PDFtoQuantiles(pred2, p=c(0.3, 0.5, 0.95), scaled=TRUE) # Gives a warning

## Demonstrate CDFtoQuantiles
X <- seq(-2, 2, length.out=300)
pdf_values <- pnorm(X)
CDFtoQuantiles(pdf_values, X, p=c(0.2, 0.5, 0.8))
qnorm(c(0.2, 0.5, 0.8))</pre>
```

predictDDConf

Prediction of Confidence Rating and Reaction Time Distribution in the drift diffusion confidence model

## **Description**

predictDDConf\_Conf predicts the categorical response distribution of decision and confidence ratings, predictDDConf\_RT computes the RT distribution (density) in the drift diffusion confidence model (Hellmann et al., 2023), given specific parameter constellations. See dDDConf for more information about the model and parameters.

#### **Usage**

```
predictDDConf_Conf(paramDf, maxrt = 15, subdivisions = 100L,
    stop.on.error = FALSE, .progress = TRUE)

predictDDConf_RT(paramDf, maxrt = 9, subdivisions = 100L, minrt = NULL,
    scaled = FALSE, DistConf = NULL, .progress = TRUE)
```

### Arguments

paramDf

a list or data frame with one row. Column names should match the names of DD-Conf model parameter names. For different stimulus quality/mean drift rates, names should be v1, v2, v3,.... Different sv and/or s parameters are possible with sv1, sv2, sv3... (s1, s2, s3,... respectively) with equally many steps as for drift rates. Additionally, the confidence thresholds should be given by names with thetaUpper1, thetaUpper2,..., thetaLower1,... or, for symmetric thresholds only by theta1, theta2,....

maxrt

numeric. The maximum RT for the integration/density computation. Default: 15 (for predictDDConf\_Conf (integration)), 9 (for predictDDConf\_RT).

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subdivisions integer (default: 100). For predictDDConf\_Conf it is used as argument for

the inner integral routine. For  ${\tt predictDDConf\_RT}$  it is the number of points for

which the density is computed.

stop.on.error logical. Argument directly passed on to integrate. Default is FALSE, since the

densities invoked may lead to slow convergence of the integrals (which are still

quite accurate) which causes R to throw an error.

. progress logical. If TRUE (default) a progress bar is drawn to the console.

minrt numeric or NULL(default). The minimum rt for the density computation.

scaled logical. For predictDDConf\_RT. Whether the computed density should be scaled

to integrate to one (additional column densscaled). Otherwise the output contains only the defective density (i.e. its integral is equal to the probability of a response and not 1). If TRUE, the argument DistConf should be given, if avail-

able. Default: FALSE.

DistConf NULL or data.frame. A data.frame or matrix with column names, giving

the distribution of response and rating choices for different conditions and stimulus categories in the form of the output of predictDDConf\_Conf. It is only necessary, if scaled=TRUE, because these probabilities are used for scaling. If scaled=TRUE and DistConf=NULL, it will be computed with the function predictDDConf\_Conf, which takes some time and the function will throw a

message. Default: NULL

#### **Details**

The function predictDDConf\_Conf consists merely of an integration of the response time density, dDDConf, over the response time in a reasonable interval (0 to maxrt). The function predictDDConf\_RT wraps these density functions to a parameter set input and a data. frame output. For the argument paramDf, the output of the fitting function fitRTConf with the DDConf model may be used.

#### Value

predictDDConf\_Conf returns a data.frame/tibble with columns: condition, stimulus, response, rating, correct, p, info, err. p is the predicted probability of a response and rating, given the stimulus category and condition. info and err refer to the respective outputs of the integration routine used for the computation. predictDDConf\_RT returns a data.frame/tibble with columns: condition, stimulus, response, rating, correct, rt and dens (and densscaled, if scaled=TRUE).

### Note

Different parameters for different conditions are only allowed for drift rate v, drift rate variability sv, and process variability s. Otherwise, s is not required in parambf but set to 1 by default. All other parameters are used for all conditions.

#### Author(s)

Sebastian Hellmann.

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#### References

Hellmann, S., Zehetleitner, M., & Rausch, M. (2023). Simultaneous modeling of choice, confidence and response time in visual perception. *Psychological Review* 2023 Mar 13. doi: 10.1037/rev0000411. Epub ahead of print. PMID: 36913292.

#### **Examples**

```
# 1. Define some parameter set in a data.frame
paramDf <- data.frame(a=2,v1=0.5, v2=1, t0=0.1,z=0.55,
                      sz=0, sv=0.2, st0=0, theta1=0.8)
# 2. Predict discrete Choice x Confidence distribution:
preds_Conf <- predictDDConf_Conf(paramDf, maxrt = 15)</pre>
head(preds_Conf)
# 3. Compute RT density
preds_RT <- predictDDConf_RT(paramDf, maxrt=4, subdivisions=200) #(scaled=FALSE)</pre>
# same output with scaled density column:
preds_RT <- predictDDConf_RT(paramDf, maxrt=4, subdivisions=200,</pre>
                              scaled=TRUE, DistConf = preds_Conf)
head(preds_RT)
 # Example of visualization
 library(ggplot2)
 preds_Conf$rating <- factor(preds_Conf$rating, labels=c("unsure", "sure"))</pre>
 preds_RT$rating <- factor(preds_RT$rating, labels=c("unsure", "sure"))</pre>
 ggplot(preds_Conf, aes(x=interaction(rating, response), y=p))+
   geom_bar(stat="identity")+
    facet_grid(cols=vars(stimulus), rows=vars(condition), labeller = "label_both")
 ggplot(preds_RT, aes(x=rt, color=interaction(rating, response), y=dens))+
   geom_line(stat="identity")+
    facet_grid(cols=vars(stimulus), rows=vars(condition), labeller = "label_both")+
    theme(legend.position = "bottom")
  ggplot(aggregate(densscaled~rt+correct+rating+condition, preds_RT, mean),
         aes(x=rt, color=rating, y=densscaled))+
    geom_line(stat="identity")+
    facet_grid(cols=vars(condition), rows=vars(correct), labeller = "label_both")+
    theme(legend.position = "bottom")
# Use PDFtoQuantiles to get predicted RT quantiles
head(PDFtoQuantiles(preds_RT, scaled = FALSE))
```

predictRM

Prediction of Confidence Rating and Reaction Time Distribution in race models of confidence

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## **Description**

predictRM\_Conf predicts the categorical response distribution of decision and confidence ratings, predictRM\_RT computes the RT distribution (density) in the independent and partially anticorrelated race models (Hellmann et al., 2023), given specific parameter constellations. See Race-Models for more information about the models and parameters.

## Usage

```
predictRM_Conf(paramDf, model = "IRM", time_scaled = FALSE, maxrt = 15,
    subdivisions = 100L, stop.on.error = FALSE, .progress = TRUE)

predictRM_RT(paramDf, model = "IRM", time_scaled = FALSE, maxrt = 9,
    subdivisions = 100L, minrt = NULL, scaled = FALSE, DistConf = NULL,
    .progress = TRUE)
```

## **Arguments**

paramDf a list or data frame with one row. Column names should match the names of

RaceModels parameter names (only mu1 and mu2 are not used in this context but replaced by the parameter v). For different stimulus quality/mean drift rates, names should be v1, v2, v3,.... Different s parameters are possible with s1, s2, s3,... with equally many steps as for drift rates. Additionally, the confidence thresholds should be given by names with thetaUpper1, thetaUpper2,...,

thetaLower1,... or, for symmetric thresholds only by theta1, theta2,....

model character scalar. One of "IRM" or "PCRM". ("IRMt" and "PCRMt" will also be

accepted. In that case, time scaled is set to TRUE.)

time\_scaled logical. Whether the confidence measure should be scaled by 1/sqrt(rt). Default:

FALSE. (It is set to TRUE, if model is "IRMt" or "PCRMt")

maxrt numeric. The maximum RT for the integration/density computation. Default:

15 (for predictRM\_Conf (integration)), 9 (for predictRM\_RT).

subdivisions integer (default: 100). For predictRM\_Conf it is used as argument for the

inner integral routine. For predictRM\_RT it is the number of points for which

the density is computed.

stop.on.error logical. Argument directly passed on to integrate. Default is FALSE, since the

densities invoked may lead to slow convergence of the integrals (which are still

quite accurate) which causes R to throw an error.

. progress logical. If TRUE (default) a progress bar is drawn to the console.

minrt numeric or NULL(default). The minimum rt for the density computation.

scaled logical. For predictRM\_RT. Whether the computed density should be scaled to

integrate to one (additional column densscaled). Otherwise the output contains only the defective density (i.e. its integral is equal to the probability of a response and not 1). If TRUE, the argument DistConf should be given, if avail-

able. Default: FALSE.

DistConf NULL or data.frame. A data.frame or matrix with column names, giving the

distribution of response and rating choices for different conditions and stimulus categories in the form of the output of predictRM\_Conf. It is only necessary, if

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scaled=TRUE, because these probabilities are used for scaling. If scaled=TRUE and DistConf=NULL, it will be computed with the function predictRM\_Conf, which takes some time and the function will throw a message. Default: NULL

#### **Details**

The function predictRM\_Conf consists merely of an integration of the response time density, dIRM and dPCRM, over the response time in a reasonable interval (0 to maxrt). The function predictRM\_RT wraps these density functions to a parameter set input and a data.frame output. For the argument paramDf, the output of the fitting function fitRTConf with the respective model may be used.

The drift rate parameters differ from those used in dIRM/dPCRM since in many perceptual decision experiments the drift on one accumulator is assumed to be the negative of the other. The drift rate of the correct accumulator is v (v1, v2, ... respectively) in paramDf.

#### Value

predictRM\_Conf returns a data.frame/tibble with columns: condition, stimulus, response, rating, correct, p, info, err. p is the predicted probability of a response and rating, given the stimulus category and condition. info and err refer to the respective outputs of the integration routine used for the computation. predictRM\_RT returns a data.frame/tibble with columns: condition, stimulus, response, rating, correct, rt and dens (and densscaled, if scaled=TRUE).

#### Note

Different parameters for different conditions are only allowed for drift rate,  $\nu$ , and process variability s. All other parameters are used for all conditions.

# Author(s)

Sebastian Hellmann.

# References

Hellmann, S., Zehetleitner, M., & Rausch, M. (2023). Simultaneous modeling of choice, confidence and response time in visual perception. *Psychological Review* 2023 Mar 13. doi: 10.1037/rev0000411. Epub ahead of print. PMID: 36913292.

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```
# 3. Compute RT density
preds_RT <- predictRM_RT(paramDf, "PCRMt", maxrt=7, subdivisions=50)</pre>
# same output with scaled density column:
preds_RT <- predictRM_RT(paramDf, "PCRMt", maxrt=7, subdivisions=50,</pre>
                         scaled=TRUE, DistConf = preds_Conf)
head(preds_RT)
 # produces a warning, if scaled=TRUE and DistConf missing
 preds_RT <- predictRM_RT(paramDf, "PCRMt", maxrt=7, subdivisions=50,</pre>
                           scaled=TRUE)
 # Example of visualization
 library(ggplot2)
 preds_Conf$rating <- factor(preds_Conf$rating, labels=c("unsure", "sure"))</pre>
 preds_RT$rating <- factor(preds_RT$rating, labels=c("unsure", "sure"))</pre>
 ggplot(preds_Conf, aes(x=interaction(rating, response), y=p))+
    geom_bar(stat="identity")+
    facet_grid(cols=vars(stimulus), rows=vars(condition), labeller = "label_both")
 ggplot(preds_RT, aes(x=rt, color=interaction(rating, response), y=dens))+
   geom_line(stat="identity")+
    facet_grid(cols=vars(stimulus), rows=vars(condition), labeller = "label_both")+
    theme(legend.position = "bottom")
  ggplot(aggregate(densscaled~rt+correct+rating+condition, preds_RT, mean),
         aes(x=rt, color=rating, y=densscaled))+
    geom_line(stat="identity")+
    facet_grid(cols=vars(condition), rows=vars(correct), labeller = "label_both")+
    theme(legend.position = "bottom")
 # Use PDFtoQuantiles to get predicted RT quantiles
 # (produces warning because of few rt steps (--> inaccurate calculations))
 PDFtoQuantiles(preds_RT, scaled = FALSE)
```

predictRTConf

Prediction of confidence rating and response time distribution for sequential sampling confidence models

# **Description**

predictConf predicts the categorical response distribution of decision and confidence ratings, predictRT computes the predicted RT distribution (density) for the sequential sampling confidence model specified by the argument model, given specific parameter constellations. This function calls the respective functions for diffusion based models (dynWEV and 2DSD: predictWEV) and race models (IRM, PCRM, IRMt, and PCRMt: predictRM).

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#### Usage

```
predictConf(paramDf, model = NULL, maxrt = 15, subdivisions = 100L,
  simult_conf = FALSE, stop.on.error = FALSE, .progress = TRUE)
predictRT(paramDf, model = NULL, maxrt = 9, subdivisions = 100L,
 minrt = NULL, simult_conf = FALSE, scaled = FALSE, DistConf = NULL,
  .progress = TRUE)
```

# Arguments

a list or dataframe with one row. Column names should match the names of paramDf

> the respective model parameters. For different stimulus quality/mean drift rates, names should be v1, v2, v3,.... Different s parameters are possible with s1, s2, s3... with equally many steps as for drift rates (same for sv parameter in dyn-WEV and 2DSD). Additionally, the confidence thresholds should be given by names with thetaUpper1, thetaUpper2,..., thetaLower1,... or, for symmetric

thresholds only by theta1, theta2,....

character scalar. One of "2DSD", "dynWEV", "IRM", "PCRM", "IRMt", or model

"PCRMt".

maxrt numeric. The maximum RT for the integration/density computation. Default:

15 (for predictConf (integration)), 9 (for predictRT).

subdivisions integer (default: 100). For predictConf it is used as argument for the inner

integral routine. For predictRT it is the number of points for which the density

is computed.

simult\_conf logical, only relevant for dynWEV and 2DSD. Whether in the experiment confi-

> dence was reported simultaneously with the decision, as then decision and confidence judgment are assumed to have happened subsequent before response and computations are different, when there is an observable interjudgment time (then

simult\_conf should be FALSE).

logical. Argument directly passed on to integrate. Default is FALSE, since the stop.on.error

densities invoked may lead to slow convergence of the integrals (which are still

quite accurate) which causes R to throw an error.

.progress logical. If TRUE (default) a progress bar is drawn to the console.

numeric or NULL(default). The minimum rt for the density computation. minrt

logical. For predictRT. Whether the computed density should be scaled to inscaled

> tegrate to one (additional column densscaled). Otherwise the output contains only the defective density (i.e. its integral is equal to the probability of a response and not 1). If TRUE, the argument DistConf should be given, if available.

Default: FALSE.

NULL or data. frame. A data. frame or matrix with column names, giving the

distribution of response and rating choices for different conditions and stimulus categories in the form of the output of predictConf. It is only necessary, if scaled=TRUE, because these probabilities are used for scaling. If scaled=TRUE and DistConf=NULL, it will be computed with the function predictRM\_Conf, which takes some time and the function will throw a message. Default: NULL

DistConf

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#### **Details**

The function predictConf consists merely of an integration of the reaction time density of the given model, {d\*model\*}, over the response time in a reasonable interval (0 to maxrt). The function predictRT wraps these density functions to a parameter set input and a data.frame output. For the argument paramDf, the output of the fitting function fitRTConf with the respective model may be used.

#### Value

predictConf returns a data.frame/tibble with columns: condition, stimulus, response, rating, correct, p, info, err. p is the predicted probability of a response and rating, given the stimulus category and condition. info and err refer to the respective outputs of the integration routine used for the computation. predictRT returns a data.frame/tibble with columns: condition, stimulus, response, rating, correct, rt and dens (and densscaled, if scaled=TRUE).

#### Note

Different parameters for different conditions are only allowed for drift rate, v, drift rate variability, sv (in dynWEV and 2DSD), and process variability s. All other parameters are used for all conditions.

#### Author(s)

Sebastian Hellmann.

```
# Examples for "dynWEV" model (equivalent applicable for
# all other models (with different parameters!))
# 1. Define some parameter set in a data.frame
paramDf \leftarrow data.frame(a=1.5,v1=0.2, v2=1, t0=0.1,z=0.52,
                       sz=0.3, sv=0.4, st0=0, tau=3, w=0.5,
                       theta1=1, svis=0.5, sigvis=0.8)
# 2. Predict discrete Choice x Confidence distribution:
preds_Conf <- predictConf(paramDf, "dynWEV", maxrt = 25, simult_conf=TRUE)</pre>
head(preds_Conf)
# 3. Compute RT density
preds_RT <- predictRT(paramDf, "dynWEV") #(scaled=FALSE)</pre>
# same output with default rt-grid and without scaled density column:
preds_RT <- predictRT(paramDf, "dynWEV", maxrt=5, subdivisions=200,</pre>
                       minrt=paramDf$tau+paramDf$t0, simult_conf = TRUE,
                       scaled=TRUE, DistConf = preds_Conf)
head(preds_RT)
  # produces a warning, if scaled=TRUE and DistConf missing
  preds_RT <- predictRT(paramDf, "dynWEV",</pre>
                            scaled=TRUE)
```

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```
# Example of visualization
 library(ggplot2)
 preds_Conf$rating <- factor(preds_Conf$rating, labels=c("unsure", "sure"))</pre>
 preds_RT$rating <- factor(preds_RT$rating, labels=c("unsure", "sure"))</pre>
 ggplot(preds_Conf, aes(x=interaction(rating, response), y=p))+
   geom_bar(stat="identity")+
   facet_grid(cols=vars(stimulus), rows=vars(condition), labeller = "label_both")
 ggplot(preds_RT, aes(x=rt, color=interaction(rating, response), y=densscaled))+
   geom_line(stat="identity")+
   facet_grid(cols=vars(stimulus), rows=vars(condition), labeller = "label_both")+
   theme(legend.position = "bottom")+ ggtitle("Scaled Densities")
 ggplot(aggregate(dens~rt+correct+rating+condition, preds_RT, mean),
         aes(x=rt, color=rating, y=dens))+
   geom_line(stat="identity")+
   facet_grid(cols=vars(condition), rows=vars(correct), labeller = "label_both")+
   theme(legend.position = "bottom")+ ggtitle("Non-Scaled Densities")
# Use PDFtoQuantiles to get predicted RT quantiles
head(PDFtoQuantiles(preds_RT, scaled = FALSE))
```

predictRTConfModels

Prediction of confidence and RT distributions for several sequential sampling confidence models and parameter constellations in parallel

# Description

This function is a wrapper around the functions predictRTConf (see there for more information). It calls the respective function for predicting the response distribution (discrete decision and rating outcomes) and the rt density (density for decision, rating and response time) for every model and participant combination in paramDf. Also, see ddynaViTE, d2DSD, and dRM for more information about the parameters.

# Usage

```
predictConfModels(paramDf, maxrt = 15, subdivisions = 100L,
    simult_conf = FALSE, stop.on.error = FALSE, .progress = TRUE,
    parallel = FALSE, n.cores = NULL)

predictRTModels(paramDf, maxrt = 9, subdivisions = 100L, minrt = NULL,
    simult_conf = FALSE, scaled = FALSE, DistConf = NULL,
    .progress = TRUE, parallel = FALSE, n.cores = NULL)
```

## **Arguments**

paramDf

a dataframe with one row per combination of model and participant/parameter set. Columns may include a participant (sbj, or subject) column, and must include a model column and the names of the model parameters. For different stimulus quality/mean drift rates, names should be v1, v2, v3,.... Different s parameters are possible with s1, s2, s3... with equally many steps as for drift rates (same for sv parameter in dynWEV and 2DSD). Additionally, the confidence thresholds should be given by names with thetaUpper1, thetaUpper2,..., thetaLower1,... or, for symmetric thresholds only by theta1, theta2,....

maxrt

numeric. The maximum RT for the integration/density computation. Default: 15 (for predictConfModels (integration)) and 9 (for predictRTModels).

subdivisions

integer (default: 100). For predictConfModels it is used as argument for the inner integral routine. For predictRTModels it is the number of points for which the density is computed.

simult\_conf

logical, only relevant for dynWEV and 2DSD. Whether in the experiment confidence was reported simultaneously with the decision, as then decision and confidence judgment are assumed to have happened subsequent before response and computations are different, when there is an observable interjudgment time (then simult\_conf should be FALSE).

stop.on.error

logical. Argument directly passed on to integrate. Default is FALSE, since the densities invoked may lead to slow convergence of the integrals (which are still quite accurate) which causes R to throw an error.

.progress

logical. If TRUE (default) a progress bar is drawn to the console. (Works for some OS only when parallel=FALSE.)

parallel

logical. If TRUE, prediction is parallelized over participants and models (i.e. over the calls for the respective predictRTConf functions).

n.cores

integer. If parallel is TRUE, the number of cores used for parallelization is required. If NULL (default) the number of available cores -1 is used.

minrt

numeric or NULL(default). The minimum rt for the density computation. If NULL, the minimal possible response time possible with given parameters will be used  $(\min(t0))$ .

scaled

logical. Whether the computed density should be scaled to integrate to one (additional column densscaled). Otherwise the output contains only the defective density (i.e. its integral is equal to the probability of a response and not 1). If TRUE, the argument DistConf should be given, if available. Default: FALSE.

DistConf

NULL or data.frame. A data.frame with participant and model columns and columns, giving the distribution of response and rating choices for different conditions and stimulus categories in the form of the output of predictConfModels. It is only necessary if scaled=TRUE, because these probabilities are used for scaling. If scaled=TRUE and DistConf=NULL, it will be computed with the function predictConfModels, which takes some time and the function will throw a message. Default: NULL

#### **Details**

These functions merely split the input data frame by model participants combinations, call the equivalent predictRTConf functions for the individual parameter sets and bind the outputs together. They are included for convenience and the easy parallelization, which facilitates speeding up computations considerably. For the argument paramDf, the output of the fitting function fitRTConfModels with the respective models and participants may be used.

The function predictConf (called by predictConfModels) consists merely of an integration of the reaction time density or the given model, {d\*model\*}, over the reaction time in a reasonable interval (0 to maxrt). The function predictRT (called by predictRTModels) wraps these density functions to a parameter set input and a data.frame output. 'Note, that the encoding for stimulus identity is different between diffusion based models (2DSD, dynWEV) and race models (IRM(t), PCRM(t)). Therefore, in the columns stimulus and response there will be a mix of encodings: -1/1 for diffusion based models and 1/2 for race models. This, usually is not important, since for further aggregation models will not be mixed.

# Value

predictConfModels returns a data.frame/tibble with columns: participant (or sbj, subject depending on the input), model, condition, stimulus, response, rating, correct, p, info, err. p is the predicted probability of a response and rating, given the stimulus category and condition. info and err refer to the respective outputs of the integration routine used for the computation. predictRTModels returns a data.frame/tibble with columns: participant (or sbj, subject depending on the input), model, condition, stimulus, response, rating, correct, rt and dens (and densscaled, if scaled=TRUE).

# Note

Different parameters for different conditions are only allowed for drift rate v, drift rate variability sv (only dynWEV and 2DSD), and process variability s. All other parameters are used for all conditions.

#### Author(s)

Sebastian Hellmann.

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```
preds_Conf <- predictConfModels(paramDf, maxrt = 15, simult_conf=TRUE,</pre>
                                 .progress=TRUE, parallel = FALSE)
# 3. Compute RT density
preds_RT <- predictRTModels(paramDf, maxrt=6, subdivisions=100,</pre>
                      scaled=TRUE, DistConf = preds_Conf,
                      parallel=FALSE, .progress = TRUE)
head(preds_RT)
 # produces a warning, if scaled=TRUE and DistConf missing
 preds_RT <- predictRTModels(paramDf, scaled=TRUE)</pre>
# Use PDFtoQuantiles to get predicted RT quantiles
head(PDFtoQuantiles(preds_RT, scaled = FALSE))
# Second Example: only one parameter set but for two different models
 paramDf1 <- data.frame(model="dynWEV", a=1.5,v1=0.2, v2=1, t0=0.1,z=0.52,</pre>
                        sz=0.3, sv=0.4, st0=0, tau=3, w=0.5,
                        theta1=1, svis=0.5, sigvis=0.8)
 paramDf2 <- data.frame(model="PCRMt", a=2,b=2, v1=0.5, v2=1, t0=0.1,st0=0,
                        wx=0.6, wint=0.2, wrt=0.2, theta1=4)
 paramDf <- dplyr::full_join(paramDf1, paramDf2)</pre>
 paramDf # each model parameters sets hat its relevant parameters
 predictConfModels(paramDf, parallel=FALSE, .progress=TRUE)
```

predictWEV

Prediction of Confidence Rating and Response Time Distribution in dynaViTE, dynWEV, and 2DSD confidence models

## **Description**

predictWEV\_Conf predicts the categorical response distribution of decision and confidence ratings, predictWEV\_RT computes the predicted RT distribution (density) in the 2DSD Model (Pleskac & Busemeyer, 2010) and the dynWEV model (Hellmann et al., 2023), given specific parameter constellations. See ddynaViTE and d2DSD for more information about parameters.

#### Usage

```
predictWEV_Conf(paramDf, model = "dynaViTE", maxrt = 15,
    subdivisions = 100L, simult_conf = FALSE, stop.on.error = FALSE,
    precision = 3, .progress = TRUE)

predictWEV_RT(paramDf, model = NULL, maxrt = 9, subdivisions = 100L,
    minrt = NULL, simult_conf = FALSE, scaled = FALSE, DistConf = NULL,
    precision = 3, .progress = TRUE)
```

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# **Arguments**

paramDf a list or dataframe with one row. Column names should match the names of

dynaViTE and 2DSD model specific parameter names. For different stimulus quality/mean drift rates, names should be v1, v2, v3,.... Different sv and/or s parameters are possible with sv1, sv2, sv3... (s1, s2, s3,... respectively) with equally many steps as for drift rates. Additionally, the confidence thresholds should be given by names with thetaUpper1, thetaUpper2,..., thetaLower1,...

or, for symmetric thresholds only by theta1, theta2,....

model character scalar. One of "dynaViTE", "dynWEV", or "2DSD".

maxrt numeric. The maximum RT for the integration/density computation. Default:

15 (for predictWEV\_Conf (integration)), 9 (for predictWEV\_RT).

subdivisions integer (default: 100). For predictWEV\_Conf it is used as argument for the

inner integral routine. For predictWEV\_RT it is the number of points for which

the density is computed.

simult\_conf logical. Whether in the experiment confidence was reported simultaneously with

the decision, as then decision and confidence judgment are assumed to have happened subsequent before response and computations are different, when there is

an observable interjudgment time (then simult\_conf should be FALSE).

stop.on.error logical. Argument directly passed on to integrate. Default is FALSE, since the

densities invoked may lead to slow convergence of the integrals (which are still

quite accurate) which causes R to throw an error.

precision numerical scalar value. Precision of calculation. Corresponds to the step size of

integration w.r.t. z and t0. Default is 1e-5.

. progress logical. if TRUE (default) a progress bar is drawn to the console.

minrt numeric or NULL(default). The minimum rt for the density computation.

scaled logical. For predictWEV\_RT. Whether the computed density should be scaled

to integrate to one (additional column densscaled). Otherwise the output contains only the defective density (i.e. its integral is equal to the probability of a response and not 1). If TRUE, the argument DistConf should be given, if avail-

able. Default: FALSE.

DistConf NULL or data.frame. A data.frame or matrix with column names, giving the

distribution of response and rating choices for different conditions and stimulus categories in the form of the output of predictWEV\_Conf. It is only necessary, if scaled=TRUE, because these probabilities are used for scaling. If scaled=TRUE and DistConf=NULL, it will be computed with the function predictWEV\_Conf, which takes some time and the function will throw a message. Default: NULL

# **Details**

The function predictWEV\_Conf consists merely of an integration of the response time density, ddynaViTE and d2DSD, over the response time in a reasonable interval (t0 to maxrt). The function predictWEV\_RT wraps these density functions to a parameter set input and a data frame output. For the argument paramDf, the output of the fitting function fitRTConf with the respective model may be used.

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#### Value

predictWEV\_Conf returns a data.frame/tibble with columns: condition, stimulus, response, rating, correct, p, info, err. p is the predicted probability of a response and rating, given the stimulus category and condition. info and err refer to the respective outputs of the integration routine used for the computation. predictWEV\_RT returns a data.frame/tibble with columns: condition, stimulus, response, rating, correct, rt and dens (and densscaled, if scaled=TRUE).

#### Note

Different parameters for different conditions are only allowed for drift rate v, drift rate variability sv, and process variability s. Otherwise, s is not required in paramDf but set to 1 by default. All other parameters are used for all conditions.

# Author(s)

Sebastian Hellmann.

#### References

Hellmann, S., Zehetleitner, M., & Rausch, M. (2023). Simultaneous modeling of choice, confidence and response time in visual perception. *Psychological Review* 2023 Mar 13. doi: 10.1037/rev0000411. Epub ahead of print. PMID: 36913292.

Pleskac, T. J., & Busemeyer, J. R. (2010). Two-Stage Dynamic Signal Detection: A Theory of Choice, Decision Time, and Confidence, *Psychological Review*, 117(3), 864-901. doi:10.1037/a0019737

```
# Examples for "dynWEV" model (equivalent applicable for "2DSD" model (with less parameters))
# 1. Define some parameter set in a data.frame
paramDf \leftarrow data.frame(a=2.5,v1=0.5, v2=1, t0=0.1,z=0.55,
                       sz=0, sv=0.2, st0=0, tau=3, w=0.3,
                       theta1=0.8, svis=0.5, sigvis=0.8)
# 2. Predict discrete Choice x Confidence distribution:
preds_Conf <- predictWEV_Conf(paramDf, "dynWEV", maxrt = 15)</pre>
head(preds_Conf)
 # To set simult_conf=TRUE makes a minor difference in the discrete distribution,
 # because we integrate over response times (we just adapt maxrt for comparison)
 preds_Conf2 <- predictWEV_Conf(paramDf, "dynWEV", simult_conf = TRUE, maxrt = 15+paramDf$tau)</pre>
 summary(preds_Conf$p-preds_Conf2$p) # difference in predicted probabilities
# 3. Compute RT density
preds_RT <- predictWEV_RT(paramDf, "dynWEV", maxrt=4, subdivisions=200) #(scaled=FALSE)</pre>
# same output with scaled density column:
preds_RT <- predictWEV_RT(paramDf, "dynWEV", maxrt=4, subdivisions=200,</pre>
                          scaled=TRUE, DistConf = preds_Conf)
head(preds_RT)
```

```
# produces a warning, if scaled=TRUE and DistConf missing
 preds_RT <- predictWEV_RT(paramDf, "dynWEV", maxrt=4, subdivisions=200,</pre>
                           scaled=TRUE)
 # Example of visualization
 library(ggplot2)
 preds_Conf$rating <- factor(preds_Conf$rating, labels=c("unsure", "sure"))</pre>
 preds_RT$rating <- factor(preds_RT$rating, labels=c("unsure", "sure"))</pre>
 ggplot(preds_Conf, aes(x=interaction(rating, response), y=p))+
   geom_bar(stat="identity")+
   facet_grid(cols=vars(stimulus), rows=vars(condition), labeller = "label_both")
 ggplot(preds_RT, aes(x=rt, color=interaction(rating, response), y=dens))+
   geom_line(stat="identity")+
   facet_grid(cols=vars(stimulus), rows=vars(condition), labeller = "label_both")+
   theme(legend.position = "bottom")
 ggplot(aggregate(densscaled~rt+correct+rating+condition, preds_RT, mean),
         aes(x=rt, color=rating, y=densscaled))+
   geom_line(stat="identity")+
   facet_grid(cols=vars(condition), rows=vars(correct), labeller = "label_both")+
   theme(legend.position = "bottom")
# Use PDFtoQuantiles to get predicted RT quantiles
head(PDFtoQuantiles(preds_RT, scaled = FALSE))
```

RaceModels

Independent and partially anti-correlated Race Model for Decision Confidence

# Description

Probability densities and random number generators for response times, decisions and confidence judgments in the independent Race Model (dIRM/rIRM) or partially (anti-)correlated Race Model (dPCRM/rPCRM), i.e. the probability of a given response (response: winning accumulator (1 or 2)) at a given time (rt) and the confidence measure in the interval between th1 and th2 (Hellmann et al., 2023). The definition of the confidence measure depends on the argument time\_scaled (see Details). The computations are based on Moreno-Bote (2010). The parameters for the models are mu1 and mu2 for the drift rates, a, b for the upper thresholds of the two accumulators and s for the incremental standard deviation of the processes and t0 and st0 for the minimum and range of uniformly distributed non-decision times (including encoding and motor time). For the computation of confidence judgments, the parameters th1 and th2 for the lower and upper bound of the interval for confidence measure and if time\_scaled is TRUE the weight parameters wx, wrt, wint for the computation of the confidence measure are required (see Details).

# Usage

```
dIRM(rt, response = 1, mu1, mu2, a, b, th1, th2, wx = 1, wrt = 0,
 wint = 0, t0 = 0, st0 = 0, s1 = 1, s2 = 1, s = NULL,
  time_scaled = TRUE, precision = 6, step_width = NULL)
dIRM2(rt, response = 1, mu1, mu2, a, b, th1, th2, wx = 1, wrt = 0,
 wint = 0, t0 = 0, st0 = 0, s1 = 1, s2 = 1, smu1 = 0, smu2 = 0,
  sza = 0, szb = 0, s = NULL, time_scaled = TRUE, precision = 6,
  step_width = NULL)
dPCRM(rt, response = 1, mu1, mu2, a, b, th1, th2, wx = 1, wrt = 0,
 wint = 0, t0 = 0, st0 = 0, s1 = 1, s2 = 1, s = NULL,
  time_scaled = TRUE, precision = 6, step_width = NULL)
rIRM(n, mu1, mu2, a, b, wx = 1, wrt = 0, wint = 0, t0 = 0, st0 = 0,
  s1 = 1, s2 = 1, s = NULL, smu1 = 0, smu2 = 0, sza = 0, szb = 0,
  time_scaled = TRUE, delta = 0.01, maxrt = 15)
rPCRM(n, mu1, mu2, a, b, wx = 1, wrt = 0, wint = 0, t0 = 0, st0 = 0,
  s1 = 1, s2 = 1, s = NULL, smu1 = 0, smu2 = 0, sza = 0, szb = 0,
  time_scaled = TRUE, delta = 0.01, maxrt = 15)
```

#### **Arguments**

rt	a numeric vector of RTs. For convenience also a data. frame with columns ${\sf rt}$ and response is possible.
response	numeric vector with values in $c(1, 2)$ , giving the accumulator that hit its boundary first.
mu1	numeric. Drift rate for the first accumulator
mu2	numeric. Drift rate for the second accumulator
a	positive numeric. Distance from starting point to boundary of the first accumulator.
b	positive numeric. Distance from starting point to boundary of the second accumulator.
th1	numeric. Lower bound of interval range for the confidence measure.
th2	numeric. Upper bound of interval range for the confidence measure.
WX	numeric. Weight on losing accumulator for the computation of the confidence measure. (Used only if time_scale=TRUE, 1)
wrt	numeric. Weight on reaction time for the computation of the confidence measure. (Used only if $time\_scale=TRUE$ , Default 0)
wint	numeric. Weight on the interaction of losing accumulator and reaction time for the computation of the confidence measure. (Used only if time_scale=TRUE, Default $0$ )
t0	numeric. Lower bound of non-decision time component in observable response times. Range: t0>=0. Default: 0.

st0	numeric. Range of a uniform distribution for non-decision time. Range: st0>=0. Default: 0.
s1	numeric. Diffusion constant of the first accumulator. Usually fixed to 1 for most purposes as it scales other parameters (see Details). Range: \$1>0, Default: 1.
s2	numeric. Diffusion constant of the second accumulator. Usually fixed to 1 for most purposes as it scales other parameters (see Details). Range: s2>0, Default: 1.
S	numeric. Alternative way to specify diffusion constants, if both are assumed to be equal. If both (s1, s2 and s) are given, only s1 and s2 will be used.
time_scaled	logical. Whether the confidence measure should be time-dependent. See Details.
precision	numerical scalar value. Precision of calculation. Determines the step size of integration w.r.t. t0. Represents the number of decimals precisely computet on average. Default is 6.
step_width	numeric. Alternative way to define the precision of integration w.r.t. t0 by directly providing the step size for the integration.
smu1	numeric. Between-trial variability in the drift rate of the first accumulator.
smu2	numeric. Between-trial variability in the drift rate of the second accumulator.
sza	numeric. Between-trial variability in starting point of the first accumulator.
szb	numeric. Between-trial variability in starting point of the second accumulator.
n	integer. The number of samples generated.
delta	numeric. Discretization step size for simulations in the stochastic process
maxrt	numeric. Maximum decision time returned. If the simulation of the stochastic process exceeds a decision time of maxrt, the response will be set to 0 and the maxrt will be returned as rt.

# **Details**

The parameters are formulated, s.t. both accumulators start at 0 and trigger a decision at their positive boundary a and b respectively. That means, both parameters have to be positive. Internally the computations adapt the parametrization of Moreno-Bote (2010).

time\_scaled determines whether the confidence measure is computed in accordance to the Balance of Evidence hypothesis (if time\_scaled=FALSE), i.e. if response is 1 at time T and  $X_2$  is the second accumulator, then

$$conf = b - X_2(T)$$

. Otherwise, if time\_scaled=TRUE (default), confidence is computed as linear combination of Balance of Evidence, decision time, and an interaction term, i.e.

$$conf = wx(b - X_2(T)) + wrt \frac{1}{\sqrt{T}} + wint \frac{b - X_2(T)}{\sqrt{T}}.$$

Usually the weights (wx, wrt, wint) should sum to 1, as the confidence thresholds (th1 and th2) may be scaled according to their sum. If this is not the case, they will be scaled accordingly internally! Usually, this formula results in lower confidence when the reaction time is longer but the

state of the second accumulator is held constant. It is based on the optimal decision confidence in Moreno-Bote (2010).

For convenience, the likelihood function allows that the first argument is a data. frame containing the information of the first and second argument in the columns (i.e., rt and response). Other columns (as well as passing response separately as argument) will be ignored.

The simulations are done by simulating normal variables in discretized steps until one process reaches the boundary. If no boundary is met within the maximum time, response is set to 0.

#### Value

dIRM and dPCRM return the numerical value of the probability density in a numerical vector of the same length as rt.

rIRM and dPCRM return a data. frame with four columns and n rows. Column names are rt (response time), response (1 or 2, indicating which accumulator hit its boundary first), x1 (the final state of the loosing accumulator), and conf (the value of the confidence measure; not discretized!).

The race parameters (as well as response, th1, and th2) are recycled to the length of the result (either rt or n). In other words, the functions are completely vectorized for all parameters and even the response.

#### Note

Similarly to the drift diffusion models (like ddiffusion and ddynaViTE), s1 and s2 are scaling factors (s1 scales: mu1 and a, s2 scales: mu2 and b, and depending on response: if response=2, s1 scales th1,th2,and wrt), otherwise s2 is the scaling factor. It is sometimes assumed (Moreno-Bote, 2010), that both noise terms are equal, then they should definitely be fixed for fitting.

# Author(s)

Sebastian Hellmann

## References

Hellmann, S., Zehetleitner, M., & Rausch, M. (2023). Simultaneous modeling of choice, confidence and response time in visual perception. *Psychological Review* 2023 Mar 13. doi: 10.1037/rev0000411. Epub ahead of print. PMID: 36913292.

Moreno-Bote, R. (2010). Decision confidence and uncertainty in diffusion models with partially correlated neuronal integrators. Neural Computation, 22(7), 1786–1811. doi:10.1162/neco.2010.12-08-930

```
# Plot rt distribution ignoring confidence curve(dPCRM(x, 1, mu1=0.5, mu2=-0.5, a=1, b=1, th1=-Inf, th2=Inf, t0=0.1), xlim=c(0,2.5)) curve(dPCRM(x, 2, mu1=0.5, mu2=-0.5, a=1, b=1, th1=-Inf, th2=Inf, t0=0.1), col="red", add=TRUE) curve(dIRM(x, 1, mu1=0.5, mu2=-0.5, a=1, b=1, th1=-Inf, th2=Inf, t0=0.1), lty=2,add=TRUE) curve(dIRM(x, 2, mu1=0.5, mu2=-0.5, a=1, b=1, th1=-Inf, th2=Inf, t0=0.1), col="red", lty=2, add=TRUE) # t0 indicates minimal response time possible abline(v=0.1)
```

```
## Following example may be equivalently used for the IRM model functions.
# Generate a random sample
df1 <- rPCRM(5000, mu1=0.2, mu2=-0.2, a=1, b=1, t0=0.1,
            wx = 1) # Balance of Evidence
# Same RT and response distribution but different confidence distribution
df2 <- rPCRM(5000, mu1=0.2, mu2=-0.2, a=1, b=1, t0=0.1,
             wint = 0.2, wrt=0.8)
head(df1)
# Compute density with rt and response as separate arguments
dPCRM(seq(0, 2, by = 0.4), response = 2, mu1 = 0.2, mu2 = -0.2, a = 1, b = 1, th1 = 0.5,
         th2=2, wx = 0.3, wint=0.4, wrt=0.1, t0=0.1)
# Compute density with rt and response in data.frame argument
df1 <- subset(df1, response !=0) # drop trials where no accumulation hit its boundary
dPCRM(df1[1:5,], mu1=0.2, mu2=-0.2, a=1, b=1, th1=0, th2=Inf, t0=0.1)
# s1 and s2 scale other decision relevant parameters
s <- 2 # common (equal) standard deviation
dPCRM(df1[1:5,], mu1=0.2*s, mu2=-0.2*s, a=1*s, b=1*s, th1=0, th2=Inf, t0=0.1, s1=s, s2=s)
s1 <- 2 # different standard deviations</pre>
s2 <- 1.5
dPCRM(df1[1:5,], mu1=0.2*s1, mu2=-0.2*s2, a=1*s1, b=1*s2, th1=0, th2=Inf, t0=0.1, s1=s1, s2=s2)
# s1 and s2 scale also confidence parameters
df1[1:5,]$response <- 2 # set response to 2
# for confidence it is important to scale confidence parameters with
# the right variation parameter (the one of the loosing accumulator)
dPCRM(df1[1:5,], mu1=0.2, mu2=-0.2, a=1, b=1,
     th1=0.5, th2=2, wx = 0.3, wint=0.4, wrt=0.1, t0=0.1)
dPCRM(df1[1:5,], mu1=0.2*s1, mu2=-0.2*s2, a=1*s1, b=1*s2,
      th1=0.5, th2=2, wx = 0.3/s1, wint = 0.4/s1, wrt = 0.1, t0=0.1, s1=s1, s2=s2)
dPCRM(df1[1:5,], mu1=0.2*s1, mu2=-0.2*s2, a=1*s1, b=1*s2,
      th1=0.5*s1, th2=2*s1, wx = 0.3, wint = 0.4, wrt = 0.1*s1, t0=0.1, s1=s1, s2=s2)
two_samples <- rbind(cbind(df1, ws="BoE"),</pre>
                   cbind(df2, ws="RT"))
# drop not finished decision processes
two_samples <- two_samples[two_samples$response!=0,]</pre>
# no difference in RT distributions
boxplot(rt~ws+response, data=two_samples)
# but different confidence distributions
boxplot(conf~ws+response, data=two_samples)
if (requireNamespace("ggplot2", quietly = TRUE)) {
require(ggplot2)
ggplot(two_samples, aes(x=rt, y=conf))+
  stat_density_2d(aes(fill = after_stat(density)),
                  geom = "raster", contour = FALSE, h=c(0.3, 0.7)) +
 xlim(c(0.2, 1.3)) + ylim(c(0, 2.5)) +
 facet_grid(cols=vars(ws), rows=vars(response), labeller = "label_both")
# Restricting to specific confidence region
df1 <- df1[df1$conf >0 & df1$conf <1,]
dPCRM(df1[1:5,], th1=0, th2=1,mu1=0.2, mu2=-0.2, a=1, b=1, t0=0.1,wx = 1)
```

rLCA 53

rLCA	Simulation of confidence ratings and RTs in leaky competing accumulator model

# Description

Simulates the decision responses, reaction times and state of the loosing accumulator together with a confidence measure in the leaky competing accumulator model. Optionally, there is a post-decisional accumulation period, where the processes continues.

# Usage

```
rLCA(n, mu1, mu2, th1, th2, k = 0, beta = 0, SPV = 0, tau = 0, wx = 1, wrt = 0, wint = 0, t0 = 0, st0 = 0, pi = 0, sig = 1, time_scaled = TRUE, simult_conf = FALSE, delta = 0.01, maxrt = 15)
```

# **Arguments**

n	integer. number of samples.
mu1	mean momentary evidence for alternative 1
mu2	mean momentary evidence for alternative 2
th1	decision threshold for alternative 1
th2	decision threshold for alternative 2
k	leakage (default: 0)
beta	inhibition (default: 0)
SPV	variation in starting points (default: 0)
tau	fixed post decisional accumulation period (default: 0)
WX	weight on balance of evidence in confidence measure (default: 1)
wrt	weight on RT in confidence measure (default: 0)
wint	weight on interaction of evidence and RT in confidence measure (default: 0)
t0	minimal non-decision time (default: 0)
st0	range of uniform distribution of non-decision time (default: 0)
pi	factor for input dependent noise of infinitesimal variance of processes (default: 0)
sig	input independent component of infinitesimal variance of processes (default: 1)
time_scaled	logical. Whether a time_scaled transformation for the confidence measure should be used.
simult_conf	logical. Whether in the experiment confidence was reported simultaneously with the decision. If that is the case decision and confidence judgment are assumed to have happened subsequent before the response. Therefore tau is included in the response time. If the decision was reported before the confidence report, simul_conf should be FALSE.

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delta numerical. Size of steps for the discretized simulation (see details).

maxrt numerical. Maximum reaction time to be simulated (see details). Default: 15.

# **Details**

The simulation is done by simulating discretized steps until one process reaches the boundary with an update rule:

$$\delta X_i(t) = \max(0, X_i(t) + \delta_t((k-1)X_i(t) - \beta X_{i=i}(t) + \mu_i + \varepsilon_i(t)),$$

with  $\varepsilon_i(t) \sim N(0, (\pi\mu_i)^2 + \sigma^2)$ . If no boundary is met within the maximum time, response is set to 0. After the decision, the accumulation continues for a time period (tau), until the final state is used for the computation of confidence.

#### Value

Returns a data. frame with three columns and n rows. Column names are rt (response time), response (1 or 2, indicating which accumulator hit its boundary first), and conf (the value of the confidence measure; not discretized!).

# Author(s)

Sebastian Hellmann.

simulateRM 55

simulateRM

Simulation of confidence ratings and RTs in race confidence models

# **Description**

Simulates the decision responses, reaction times and state of the loosing accumulator together with a discrete confidence judgment in the independent and partially anti-correlated race model (IRM and PCRM) (Hellmann et al., 2023), given specific parameter constellations. See RaceModels for more information about parameters. Also computes the Gamma rank correlation between the confidence ratings and condition (task difficulty), reaction times and accuracy in the simulated output. Basically, this function is a wrapper for rIRM and rPCRM for application in confidence experiments with manipulation of specific parameters. rRM\_Kiani simulates a different version of race models, presented in Kiani et al. (2014), but without a confidence measure.

# Usage

```
simulateRM(paramDf, n = 10000, model = "IRM", time_scaled = FALSE,
  gamma = FALSE, agg_simus = FALSE, stimulus = c(1, 2), delta = 0.01,
  maxrt = 15, seed = NULL)

rRM_Kiani(paramDf, n = 10000, time_scaled = FALSE, gamma = FALSE,
  agg_simus = FALSE, stimulus = c(1, 2), delta = 0.01, maxrt = 15,
  seed = NULL)
```

# **Arguments**

paramDf	a list or data frame with one row. Column names should match the names of RaceModels parameter names (only mu1 and mu2 are not used in this context but replaced by the parameter v). For different stimulus quality/mean drift rates, names should be v1, v2, v3, Different s parameters are possible with s1, s2, s3, with equally many steps as for drift rates. Additionally, the confidence thresholds should be given by names with thetaUpper1, thetaUpper2,, thetaLower1, or, for symmetric thresholds only by theta1, theta2,
n	integer. The number of samples (per condition and stimulus direction) generated. Total number of samples is $n*nConditions*length(stimulus)$ .
model	character scalar. One of "IRM" or "PCRM". ("IRMt" and "PCRMt" will also be accepted. In that case, time_scaled is set to TRUE.)
time_scaled	logical. Whether a time_scaled transformation for the confidence measure should be used.
gamma	logical. If TRUE, the gamma correlation between confidence ratings, rt and accuracy is computed.
agg_simus	logical. Simulation is done on a trial basis with RTs outcome. If TRUE, the simulations will be aggregated over RTs to return only the distribution of response and confidence ratings. Default: FALSE.

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numeric vector. Either 1, 2 or c(1, 2) (default). Together with condition represents the experimental situation. In a binary decision task the presented stimulus belongs to one of two categories. In the default setting trials with both categories presented are simulated but one can choose to simulate only trials with the stimulus coming from one category (each associated with positive drift in one of two

accumulators).

delta numerical. Size of steps for the discretized simulation (see details).

maxrt numerical. Maximum reaction time to be simulated (see details). Default: 15.
seed numerical. Seeding for non-random data generation. (Also possible outside of

the function.)

#### **Details**

The simulation is done by simulating normal variables in discretized steps until one process reaches the boundary. If no boundary is met within the maximum time, response is set to 0. The output of the fitting function fitRTConf with the respective model fits the argument paramDf for simulation. The Gamma coefficients are computed separately for correct/incorrect responses for the correlation of confidence ratings with condition and rt and separately for conditions for the correlation of accuracy and confidence. The resulting data frames in the output thus have two columns. One for the grouping variable and one for the Gamma coefficient.

#### Value

Depending on gamma and agg\_simus.

If gamma is FALSE, returns a data. frame with columns: condition, stimulus, response, correct, rt, conf (the continuous confidence measure) and rating (the discrete confidence rating) or (if agg\_simus=TRUE): condition, stimulus,response, correct, rating and p (for the probability of a response and rating, given the condition and stimulus).

If gamma is TRUE, returns a list with elements: simus (the simulated data frame) and gamma, which is again a list with elements condition, rt and correct, each a tibble with two columns (see details for more information).

#### Note

Different parameters for different conditions are only allowed for drift rate, v, and process variability, s. All other parameters are used for all conditions.

# Author(s)

Sebastian Hellmann.

# References

Hellmann, S., Zehetleitner, M., & Rausch, M. (2023). Simultaneous modeling of choice, confidence and response time in visual perception. *Psychological Review* 2023 Mar 13. doi: 10.1037/rev0000411. Epub ahead of print. PMID: 36913292.

Kiani, R., Corthell, L., & Shadlen, M.N. (2014) Choice certainty is informed by both evidence and decision time. Neuron, 84(6), 1329-1342. doi:10.1016/j.neuron.2014.12.015

simulateRTConf 57

# **Examples**

```
# Examples for "PCRM" model (equivalent applicable for "IRM" model)
# 1. Define some parameter set in a data.frame
paramDf <- data.frame(a=2,b=2, v1=0.5, v2=1, t0=0.1,st0=0,
                      wx=0.6, wint=0.2, wrt=0.2,
                      theta1=4)
# 2. Simulate trials for both stimulus categories and all conditions (2)
simus <- simulateRM(paramDf, n=30,model="PCRM", time_scaled=TRUE)</pre>
head(simus)
# equivalent:
simus <- simulateRM(paramDf, model="PCRMt")</pre>
 library(ggplot2)
 simus <- simus[simus$response!=0,]</pre>
 simus$rating <- factor(simus$rating, labels=c("unsure", "sure"))</pre>
 ggplot(simus, aes(x=rt, group=interaction(correct, rating),
                    color=as.factor(correct), linetype=rating))+
    geom_density(size=1.2)+
    facet_grid(rows=vars(condition), labeller = "label_both")
# automatically aggregate simulation distribution
# to get only accuracy x confidence rating distribution for
# all conditions
agg_simus <- simulateRM(paramDf, n = 20, model="PCRMt", agg_simus = TRUE)
head(agg_simus)
 agg_simus$rating <- factor(agg_simus$rating, labels=c("unsure", "sure"))</pre>
 library(ggplot2)
 ggplot(agg_simus, aes(x=rating, group=correct, fill=as.factor(correct), y=p))+
    geom_bar(stat="identity", position="dodge")+
    facet_grid(cols=vars(condition), labeller = "label_both")
 # Compute Gamma correlation coefficients between
 # confidence and other behavioral measures
 # output will be a list
 simu_list <- simulateRM(paramDf, model="IRMt", gamma=TRUE)</pre>
 simu_list
```

simulateRTConf

Simulation of confidence ratings and RTs in sequential sampling confidence models

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# **Description**

Simulates the decision responses, reaction times and confidence measure together with a discrete confidence judgment for the sequential sampling confidence model specified by the argument model, given specific parameter constellations. This function is a wrapper that calls the respective functions for diffusion based models (dynaViTE and 2DSD: simulateWEV) and race models (IRM, PCRM, IRMt, and PCRMt: simulateRM. It also computes the Gamma rank correlation between the confidence ratings and condition (task difficulty), reaction times and accuracy in the simulated output.

# Usage

```
simulateRTConf(paramDf, n = 10000, model = NULL, gamma = FALSE,
  agg_simus = FALSE, simult_conf = FALSE, stimulus = c(1, 2),
  delta = 0.01, maxrt = 15, seed = NULL)
```

# **Arguments**

•		
	paramDf	a list or dataframe with one row with the required parameters.
	n	integer. The number of samples (per condition and stimulus direction) generated. Total number of samples is $n*nConditions*length(stimulus)$ .
	model	character scalar. One of "dynaViTE", "dynWEV", "2DSD", "2DSDT", "IRM", "PCRM", "IRMt", or "PCRMt". Could also be passed as a column in the paramDf argument.
	gamma	logical. If TRUE, the gamma correlation between confidence ratings, rt and accuracy is computed.
	agg_simus	logical. Simulation is done on a trial basis with RTs outcome. If TRUE, the simulations will be aggregated over RTs to return only the distribution of response and confidence ratings. Default: FALSE.
	simult_conf	logical. Whether in the experiment confidence was reported simultaneously with the decision. If that is the case decision and confidence judgment are assumed to have happened subsequent before the response. Therefore tau is included in the response time. If the decision was reported before the confidence report, simul_conf should be FALSE.
	stimulus	numeric vector. Either 1, 2 or $c(1, 2)$ (default). Together with condition represents the experimental situation. In a 2AFC task the presented stimulus belongs to one of two categories. In the default setting trials with both categories presented are simulated but one can choose to simulate only trials with the stimulus coming from one category.
	delta	numerical. Size of steps for the discretized simulation.
	maxrt	numerical. Maximum reaction time to be simulated. Default: 15.
	seed	numerical. Seeding for non-random data generation. (Also possible outside of the function.)

## **Details**

The output of the fitting function fitRTConf with the respective model fits the argument paramDf for simulation. The function calls the respective simulation function for diffusion based models, i.e.

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dynaViTE and 2DSD (simulateWEV) or race models, i.e. IRM(t) and PCRM(t), (simulateRM). See there for more information.

**Simulation Method:** The simulation is done by simulating normal variables in discretized steps until the processes reach the boundary. If no boundary is met within the maximum time, response is set to 0.

**Gamma correlations:** The Gamma coefficients are computed separately for correct/incorrect responses for the correlation of confidence ratings with condition and rt and separately for conditions for the correlation of accuracy and confidence. The resulting data frames in the output thus have two columns. One for the grouping variable and one for the Gamma coefficient.

# Value

Depending on gamma and agg\_simus.

If gamma is FALSE, returns a data. frame with columns: condition, stimulus, response, correct, rt, conf (the continuous confidence measure) and rating (the discrete confidence rating) or (if agg\_simus=TRUE): condition, stimulus,response, correct, rating and p (for the probability of a response and rating, given the condition and stimulus).

If gamma is TRUE, returns a list with elements: simus (the simulated data frame) and gamma, which is again a list with elements condition, rt and correct, each a tibble with two columns (see details for more information).

## Author(s)

Sebastian Hellmann.

```
# The function is particularly useful, when having a collection
# of parameter sets for different models (e.g. output by fitRTConfModels for
# more than one model).
library(dplyr)
# 1. Generate only one parameter set but for two different models
paramDf1 <- data.frame(model="dynWEV", a=1.5,v1=0.2, v2=1, t0=0.1,z=0.52,</pre>
                      sz=0.3,sv=0.4, st0=0, tau=3, w=0.5,
                      theta1=1, svis=0.5, sigvis=0.8)
paramDf2 <- data.frame(model="PCRMt", a=2,b=2, v1=0.5, v2=1, t0=0.1,st0=0,
                      wx=0.6, wint=0.2, wrt=0.2, theta1=4)
paramDf <- full_join(paramDf1, paramDf2)</pre>
paramDf # each model parameters sets hat its relevant parameters
# Split paramDf by model (maybe also other columns) and simulate data
simus <- paramDf |> group_by(model) |>
reframe(simulateRTConf(cbind(cur_group(), pick(everything())), n=200, simult_conf = TRUE))
head(simus)
```

60 simulateWEV

simulateWEV	Simulation of confidence ratings and RTs in dynWEV and 2DSD confidence models

# **Description**

Simulates the decision responses and reaction times together with a discrete confidence judgment in the dynaViTE model, the 2DSD model (Pleskac & Busemeyer, 2010) and the dynWEV model (Hellmann et al., 2023), given specific parameter constellations. See ddynaViTE and d2DSD for more information about parameters. Also computes the Gamma rank correlation between the confidence ratings and condition (task difficulty), reaction times and accuracy in the simulated output. Basically, this function is a wrapper for rdynaViTE and r2DSD for application in confidence experiments with manipulation of specific parameters.

# Usage

```
simulateWEV(paramDf, n = 10000, model = "dynWEV", simult_conf = FALSE,
gamma = FALSE, agg_simus = FALSE, stimulus = c(-1, 1), delta = 0.01,
maxrt = 15, seed = NULL, process_results = FALSE)
```

# **Arguments**

paramDf	a list or dataframe with one row. Column names should match the names of dynaViTE and 2DSD model specific parameter names. For different stimulus quality/mean drift rates, names should be v1, v2, v3, Different sv and/or s parameters are possible with sv1, sv2, sv3 (s1, s2, s3, respectively) with equally many steps as for drift rates. Additionally, the confidence thresholds should be given by names with thetaUpper1, thetaUpper2,, thetaLower1, or, for symmetric thresholds only by theta1, theta2,
n	integer. The number of samples (per condition and stimulus direction) generated. Total number of samples is n*nConditions*length(stimulus).
model	character scalar. One of "dynaViTE", "dynWEV", or "2DSD".
simult_conf	logical. TRUE, if in the experiment confidence was reported simultaneously with the decision, as then decision and confidence judgment are assumed to have happened subsequent before response and tau is added to the simulated decision time. If FALSE returned response time will only be decision time plus non-judgment time component.
gamma	logical. If TRUE, the gamma correlation between confidence ratings, rt and accuracy is computed.
agg_simus	logical. Simulation is done on a trial basis with RTs outcome. If TRUE, the simulations will be aggregated over RTs to return only the distribution of response and confidence ratings. Default: FALSE.
stimulus	numeric vector. Either 1, -1 or c(-1, 1) (default). Together with condition represents the experimental situation. In a binary decision task the presented stimulus belongs to one of two categories. In the default setting trials with both categories

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presented are simulated but one can choose to simulate only trials with the stimulus coming from one category (1 for the category that is associated with positive drift in the decision process where "upper"/1 responses are considered correct and -1 correspondingly for negative drifts and "lower"/-1 correct decisions).

delta numeric. Discretization steps for simulations with the stochastic process.

maxrt numeric. Maximum reaction time returned. If the simulation of the stochastic

process exceeds a rt of maxrt, the response will be set to 0 and maxrt will be

returned as rt.

seed numerical. Seeding for non-random data generation.

process\_results

logical. Whether the output simulations should contain the final state of the decision (and visibility) process as additional column. Default is FALSE, meaning that no additional columns for the final process states are returned.

#### **Details**

Simulation of response and decision times is done by simulating normal variables in discretized steps until the lower or upper boundary is met (or the maximal rt is reached). Afterwards, a confidence measure is simulated according to the respective model.

The confidence outputs are then binned according to the given thresholds. The output of the fitting function fitRTConf with the respective model fits the argument paramDf for simulation. The Gamma coefficients are computed separately for correct/incorrect responses for the correlation of confidence ratings with condition and rt and separately for conditions for the correlation of accuracy and confidence. The resulting data frames in the output thus have two columns. One for the grouping variable and one for the Gamma coefficient.

#### Value

Depending on gamma and agg\_simus.

If gamma is FALSE, returns a data. frame with columns: condition, stimulus, response, correct, rt, conf (the continuous confidence measure) and rating (the discrete confidence rating), and dec and vis (only if process\_results=TRUE) for the final states of accumulators in the simulation or (if agg\_simus=TRUE): condition, stimulus,response, correct, rating and p (for the probability of a response and rating, given the condition and stimulus).

If gamma is TRUE, returns a list with elements: simus (the simulated data frame) and gamma, which is again a list with elements condition, rt and correct, each a tibble with two columns (see details for more information).

#### Note

Different parameters for different conditions are only allowed for drift rate, v, drift rate variability, sv and diffusion constant s. All other parameters are used for all conditions.

#### Author(s)

Sebastian Hellmann.

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#### References

Hellmann, S., Zehetleitner, M., & Rausch, M. (2023). Simultaneous modeling of choice, confidence and response time in visual perception. *Psychological Review* 2023 Mar 13. doi: 10.1037/rev0000411. Epub ahead of print. PMID: 36913292.

```
# Examples for "dynWEV" model (equivalent applicable
# for "2DSD" model (with less parameters))
# 1. Define some parameter set in a data.frame
paramDf <- data.frame(a=2.5,v1=0.1, v2=1, t0=0.1,z=0.55,
                      sz=0.3,sv=0.8, st0=0, tau=3, w=0.1,
                      theta1=0.8, svis=0.5, sigvis=0.8)
# 2. Simulate trials for both stimulus categories and all conditions (2)
simus <- simulateWEV(paramDf, model="dynWEV")</pre>
head(simus)
 library(ggplot2)
 simus <- simus[simus$response!=0,]</pre>
 simus$rating <- factor(simus$rating, labels=c("unsure", "sure"))</pre>
 ggplot(simus, aes(x=rt, group=interaction(correct, rating),
                    color=as.factor(correct), linetype=rating))+
    geom_density(size=1.2)+xlim(c(0,5))+
   facet_grid(rows=vars(condition), labeller = "label_both")
# automatically aggregate simulation distribution
# to get only accuracy x confidence rating distribution for
# all conditions
agg_simus <- simulateWEV(paramDf, model="dynWEV", agg_simus = TRUE)</pre>
head(agg_simus)
 agg_simus$rating <- factor(agg_simus$rating, labels=c("unsure", "sure"))</pre>
 library(ggplot2)
 ggplot(agg_simus, aes(x=rating, group=correct, fill=as.factor(correct), y=p))+
   geom_bar(stat="identity", position="dodge")+
   facet_grid(cols=vars(condition), labeller = "label_both")
 # Compute Gamma correlation coefficients between
 # confidence and other behavioral measures
 # output will be a list
 simu_list <- simulateWEV(paramDf,n = 400, model="dynWEV", gamma=TRUE)</pre>
 simu_list
```

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